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NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

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LAMINAR FLOW CIRCULATION IN A ROTATING TANK WITH A SPINNING COVER

Hans J. Lugt, Henry J. Haussling, Samuel Ohring

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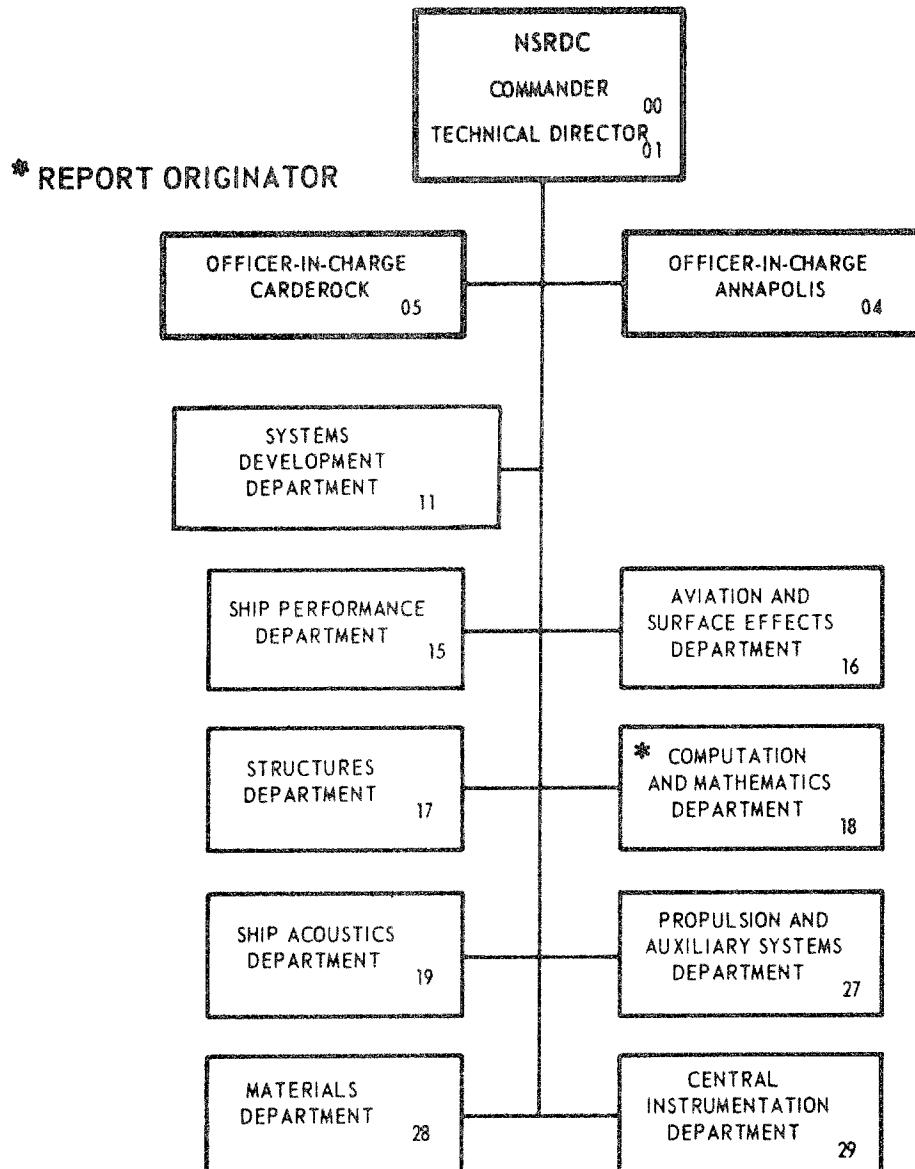
Report 3797

LAMINAR FLOW CIRCULATION IN A ROTATING TANK WITH A SPINNING COVER

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ROTATING TANK WITH A
SPINNING COVER

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NOTATION

A, B, C, D, E	Abbreviations introduced in Equation (18)
a, b	Stretching factors defined by Equation (15)
c	Constant defined in Equation (29)
Ek	Ekman number
f	Frequency
H	Height of the tank
k	Summation index
\vec{k}	Unit vector in the direction of rotation
L	Radius of the tank
l, m	Eigenvalues
p'	Pressure
p	Dimensionless pressure
Ro	Rossby number
r', ϕ, z'	Cylindrical polar coordinates
r, ϕ, z	Dimensionless cylindrical polar coordinates
t'	Time
t	Dimensionless time
u', v', w'	Velocity components corresponding to r', ϕ, z'
u, v, w	Dimensionless velocity components
α, β	Abbreviations introduced in Equation (18)
γ_k	Abbreviation introduced in Equation (28)
δ	Ratio H/L
$\vec{\zeta}$	Dimensionless vorticity vector
ζ	Azimuthal component of $\vec{\zeta}$
η, θ	Stretched coordinates z, r

ν	Kinematic viscosity
ρ	Density
σ, τ	Abbreviations introduced in Equation (28)
ψ	Dimensionless stream function
χ_k	Function introduced in Equation (28)
Ω	Angular velocity of the tank
Ω_c	Angular velocity of the cover
ω	$= \Omega - \Omega_c$

Sub- and Superscripts:

i, j	Location of grid point in the (η, θ) -plane
n	Location of grid point in time

ABSTRACT

A study has been made of axisymmetric incompressible fluid flows in a rotating tank when the angular speed of the cover changes abruptly. From the initial solid-body rotation a meridional and an azimuthal circulation relative to the moving tank develop. This problem is solved numerically by means of a stream function-vorticity formulation for the meridional flow. Local fine grids are used in the Ekman and Stewartson layers. No finite gap between tank and cover is considered. The singular behavior at this point is investigated. The parameters considered are the Rossby number, the Ekman number, and the ratio of height to radius of the tank. Temporal and spatial oscillations of the laminar flow field as well as the occurrence of cell flows are discussed.

ADMINISTRATIVE INFORMATION

This study was supported by the Naval Ship Systems Command under the Mathematical Sciences Program, Subproject SR 003 03 01. An abridged version was presented at the IUTAM Symposium on Unsteady Boundary Layers, Laval University, Quebec, Canada, 25-29 May 1971 and was published in the Proceedings of the Symposium under the title "Transient Ekman and Stewartson Layers in a Rotating Tank with a Spinning Cover."

1. INTRODUCTION

In recent years the study of rotating fluids has found widespread interest from both the theoretical and practical point of view. Applications of this subject range from lubrication problems to centrifuge design to geophysical questions. Physically, rotating fluids can behave quite differently from nonrotating ones, a fact which has important mathematical implications. In the Computation and Mathematics Department the program to simulate viscous fluid flows by means of computers includes the study of rotating fluids. The present analysis is an outgrowth of this effort.

We consider the following laminar flow problem. A circular-cylindrical tank, completely filled with an incompressible fluid, is rotating around its axis with a constant angular velocity. The cover is a disk which can rotate co-axially with the tank but at a different rate. Initially, both tank and cover rotate with the same angular velocity, and the fluid inside behaves as a solid body. At a certain instant, the angular speed of the cover abruptly changes to a rate which is different from that of the tank. As a result, a meridional and an azimuthal circulation relative to the moving tank develop with time and approach asymptotically a steady state. A numerical computation of this flow is the subject of the present study.

A number of finite-difference solutions, obtained with the stream function-vorticity formulation for the meridional flow, have recently been published in the literature. Steady-state integrals for flows within a fixed tank with a spinning cover were found by Dorfman and Romanenko¹ and by Pao². The steady motions in a

¹ References are listed on page 43.

rotating tank with a fixed cover were studied by Pao³ and by Farris et al⁴. Pao³ also computed the initial phase of the flows after the abrupt change of the cover's rotational speed. Flows with sources and sinks on the boundaries were investigated by Farris et al⁴ for the steady-state case and by Krause⁵ for the transient case. Small deviations from solid-body rotation allow the linearization of the equations of motion. This approach was chosen by Rasmussen⁶. The spin-up and spin-down of a cylindrical tank without relative rotation of the cover was studied by Briley and Walls⁷.

A wealth of literature exists for limiting cases which permit similarity assumptions or the use of perturbation methods. The solutions obtained are valuable as a means of checking the accuracy of the numerical techniques. Older papers are recorded in Schlichting's book⁸, newer papers in Greenspan's monograph⁹. In particular, attention is called to the paper by Pearson¹⁰. The spin-up and spin-down problems were studied by Greenspan⁹ and by Euteneuer et al¹¹. Details of some of the papers are discussed in context with our results.

The problem outlined in the beginning of this section is solved by means of a stream function-vorticity formulation. The numerical technique is essentially that presented by the authors in Reference 12. Grid systems of $41 \times 41 = 1681$ and $51 \times 51 = 2601$ mesh points with unequal spacing represent the flow field. Thus, local fine grids can be used in regions with high vorticity gradients. The parameters of the problem are the Rossby number Ro , Ekman number Ek , and the aspect ratio δ of the tank.

The assignment of values to these parameters is guided by the objective of studying flows in a tank with nonvanishing angular speed. It is of advantage to solve the equations of motion in a rotating frame

when studying the elliptic or hyperbolic flow behavior in space or in time for small Rossby numbers. This can be seen immediately for the linearized case, $Ro = 0$. Then the nondimensional vorticity transport equation takes the form

$$\left(\frac{\partial^2}{\partial t^2} - 2Ek \frac{\partial}{\partial t} \nabla^2 + Ek^2 \nabla^4 \right) \nabla^2 \vec{\zeta} + (2\vec{k} \cdot \nabla)^2 \vec{\zeta} = 0,$$

where \vec{k} is the unit vector in the direction of rotation. The other quantities are described in Section 2. When Ek is small the hyperbolic form of the equation in time is revealed. For steady motions we arrive at the Taylor-Proudman theorem if $Ek = 0^9$.

2. FORMULATION OF THE INITIAL-BOUNDARY VALUE PROBLEM

We assume a laminar axisymmetric flow of an incompressible fluid in a circular-cylindrical tank of radius L and height H , which is spinning with constant angular velocity Ω . At time $t'=0$ the cover impulsively starts to rotate with a different but constant angular velocity Ω_c . Cylindrical polar coordinates r' , ϕ , z' are used with the corresponding velocity components u' , v' , w' in a reference frame rotating with the tank. Under the restriction of axisymmetry, $\partial/\partial\phi \equiv 0$, the Navier-Stokes equations and the equation of continuity are

$$u'_{t'} + u'u'_{r'} + w'u'_{z'} - v'(2\Omega + \frac{v'}{r'}) = -\frac{1}{\rho} p'_{r'} + \nu [u'_{r'r'} + (\frac{u'}{r'})_{r'} + u'_{z'z'}], \quad (1)$$

$$v'_{t'} + u'v'_{r'} + w'v'_{z'} + u'(2\Omega + \frac{v'}{r'}) = \nu [v'_{r'r'} + (\frac{v'}{r'})_{r'} + v'_{z'z'}], \quad (2)$$

$$w'_t + u'w'_r + w'w'_{zz} = -\frac{1}{\rho} p'_{zz} + \nu [w'_{rr} + \frac{w'_r}{r'} + w'_{zz}], \quad (3)$$

$$u'_{rr} + \frac{u'_r}{r'} + w'_{zz} = 0. \quad (4)$$

Here, p' , ρ , and ν are the pressure, the constant density, and the constant kinematic viscosity, respectively. Prior to the sudden change of rotation of the cover, the entire fluid is at rest relative to the spinning tank:

$$t' < 0: \quad u' \equiv 0, \quad v' \equiv 0, \quad w' \equiv 0. \quad (5)$$

After the change of rotation, $t' \geq 0$, the boundary conditions are

$$\begin{aligned} z' = 0, \quad 0 \leq r' \leq L: \quad u' = 0, \quad v' = 0, \quad w' = 0, \\ z' = H, \quad 0 \leq r' \leq L: \quad u' = 0, \quad v' = -\omega r', \quad w' = 0, \\ r' = L, \quad 0 \leq z' < H: \quad u' = 0, \quad v' = 0, \quad w' = 0, \end{aligned} \quad (6)$$

where $\omega = \Omega - \Omega_c$. It is convenient to introduce the following dimensionless variables:

$$t' = t/\Omega, \quad r' = Lr, \quad z' = Hz, \quad (u', v') = \omega L(u, v), \quad w' = \omega H w, \quad p' = \rho \omega \Omega L^2 p \quad (7)$$

and the characteristic numbers

$$Ro = \frac{\omega}{\Omega} \quad (\text{Rossby number}),$$

$$Ek = \frac{\nu}{\Omega H^2} \quad (\text{Ekman number}), \quad (8)$$

$$\delta = \frac{H}{L} \quad (\text{aspect ratio}),$$

The axisymmetry of the motion permits the stream function-vorticity formulation of the meridional flow. If ψ designates the dimensionless stream function and ζ the azimuthal component of the dimensionless

vorticity vector $\vec{\zeta}$, where

$$u = \frac{1}{r} \psi_z, \quad w = -\frac{1}{r} \psi_r, \quad (9)$$

$$\zeta = u_z - \delta^2 w_r, \quad (10)$$

Equations (1) through (4) are reduced to

$$\zeta_t + \text{Ro}[(u\zeta)_r + (w\zeta)_z - (\frac{v^2}{r})_z] - 2v_z = \text{Ek}[\delta^2(\zeta_{rr} + \frac{1}{r}\zeta_r - \frac{1}{r^2}\zeta) + \zeta_{zz}], \quad (11)$$

$$v_t + \text{Ro}[(uv)_r + (wv)_z + 2\frac{uv}{r}] + 2u = \text{Ek}[\delta^2(v_{rr} + \frac{1}{r}v_r - \frac{v}{r^2}) + v_{zz}], \quad (12)$$

$$\frac{1}{r}[\delta^2(\psi_{rr} - \frac{1}{r}\psi_r) + \psi_{zz}] = \zeta. \quad (13)$$

The corresponding boundary conditions for $t \geq 0$ are

$$\begin{aligned} z = 0, \quad 0 \leq r \leq 1: \quad \psi = 0, \quad \psi_z = 0, \quad v = 0, \\ z = 1, \quad 0 \leq r \leq 1: \quad \psi = 0, \quad \psi_z = 0, \quad v = -r, \\ r = 1, \quad 0 \leq z \leq 1: \quad \psi = 0, \quad \psi_r = 0, \quad v = 0, \\ r = 0, \quad 0 \leq z < 1: \quad \psi = 0, \quad \zeta = 0, \quad v = 0. \end{aligned} \quad (14)$$

The last conditions for the values on the centerline follow from the axisymmetry of the flow, which allows restriction to half the meridional plane.

At the corner $r = 1, z = 1$ a discontinuity in v occurs which causes the shear stresses $\lim_{r \rightarrow 1} \nu \rho \left(\frac{\partial v}{\partial z} \right)_{z=1}$ and $\lim_{z \rightarrow 1} \nu \rho \left(\frac{\partial v}{\partial r} \right)_{r=1}$ to be unbounded at that point. In reality, a small gap between cover and container always exists. Thus, this singularity is avoided. The matter was discussed by Schmieden¹³ for slow motion with no

meridional flow. If the gap is taken to be infinitesimal (as in this work), the torques exerted on the cover and the tank are always logarithmically singular. Although the singularity becomes more and more localized with decreasing Ekman number, the torques are always infinite. The implications for the finite-difference scheme are discussed below.

3. NUMERICAL INTEGRATION

For finite-difference techniques it is of advantage to lay out a network which is dense in regions of high vorticity gradients. Such regions usually occur as boundary layers near solid walls. For the present problem the following coordinate transformation is introduced:

$$\begin{aligned} r &= \theta + a \sin \pi \theta, \\ z &= \eta - b \sin 2\pi \eta, \end{aligned} \tag{15}$$

where the stretching factors a and b are chosen from the intervals $0 \leq a \leq 0.26$ and $0.10 \leq b \leq 0.13$. The region of integration is represented by a grid with mesh points at $\eta = \eta_i = (i-1)\Delta\eta$, $\theta = \theta_j = (j-1)\Delta\theta$, where $i, j = 1, \dots, 41$ or 51 . (See Figure 1.)

After the equations of motion are transformed according to Equation (15), they are solved with the following finite-difference scheme. The linear differential operators of Equations (11) and (12), except for the Coriolis terms, are replaced by the Dufort-Frankel approximation. The term $-2v_z$ and the nonlinear operators are expressed by central-difference formulae. Then, Equations (11)

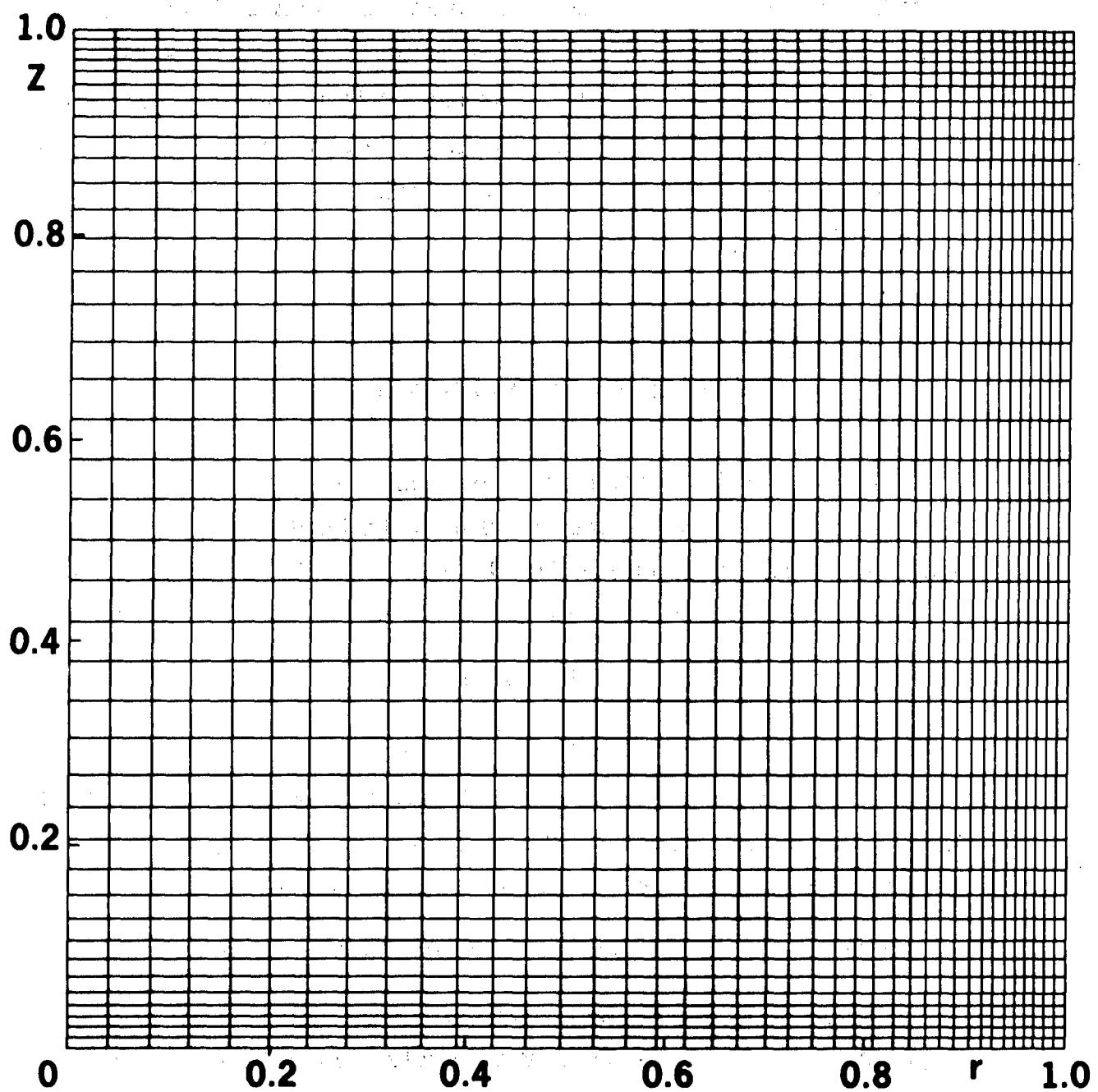


Figure 1 - Grid System with 41×41 Mesh Points
and Stretching Factors $a = 0.2$, $b = 0.1$

and (12) yield, for the $(n+1)^{\text{th}}$ time step,

$$\begin{aligned}
 \zeta_{i,j}^{n+1} = & (1 - 2E_k \Delta t C_{i,j})^{-1} \{ \zeta_{i,j}^{n-1} + 2\Delta t [\text{Ro}(\alpha_j) \frac{u_{i,j-1}^n \zeta_{i,j-1}^n - u_{i,j+1}^n \zeta_{i,j+1}^n}{2\Delta\theta} \\
 & + \beta_i \frac{w_{i-1,j}^n \zeta_{i-1,j}^n - w_{i+1,j}^n \zeta_{i+1,j}^n}{2\Delta\eta} + \frac{v_{i,j}^n}{r_j} \beta_i \frac{v_{i+1,j}^n - v_{i-1,j}^n}{\Delta\eta}) \\
 & + \beta_i \frac{v_{i+1,j}^n - v_{i-1,j}^n}{\Delta\eta} + E_k (A_j \zeta_{i,j+1}^n + B_j \zeta_{i,j-1}^n + C_{i,j} \zeta_{i,j}^{n-1} \\
 & + D_i \zeta_{i+1,j}^n + E_i \zeta_{i-1,j}^n)] \} , \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 v_{i,j}^{n+1} = & (1 - 2E_k \Delta t C_{i,j})^{-1} \{ v_{i,j}^{n-1} + 2\Delta t [\text{Ro}(\alpha_j) \frac{u_{i,j-1}^n v_{i,j-1}^n - u_{i,j+1}^n v_{i,j+1}^n}{2\Delta\theta} \\
 & + \beta_i \frac{w_{i-1,j}^n v_{i-1,j}^n - w_{i+1,j}^n v_{i+1,j}^n}{2\Delta\eta} - \frac{2}{r_j} u_{i,j}^n v_{i,j}^n) - 2u_{i,j} \\
 & + E_k (A_j v_{i,j+1}^n + B_j v_{i,j-1}^n + C_{i,j} v_{i,j}^{n-1} + D_i v_{i+1,j}^n + E_i v_{i-1,j}^n)] \} . \tag{17}
 \end{aligned}$$

Here, the following abbreviations are used:

$$\alpha_j = \frac{1}{1 + a\pi \cos \pi \theta_j} , \quad \beta_i = \frac{1}{1 - 2\pi b \cos 2\pi \eta_i} ,$$

$$r_j = \theta_j + a \sin \pi \theta_j ,$$

$$A_j = \left(\frac{\delta \alpha_j}{\Delta \theta} \right)^2 + \frac{\pi^2 \delta^2 \alpha_j^3 a \sin \pi \theta_j}{2 \Delta \theta} + \frac{\delta^2 \alpha_j}{2 r_j \Delta \theta} ,$$

$$B_j = \left(\frac{\delta \alpha_j}{\Delta \theta} \right)^2 - \frac{\pi^2 \delta^2 \alpha_j^3 a \sin \pi \theta_j}{2 \Delta \theta} - \frac{\delta^2 \alpha_j}{2 r_j \Delta \theta} ,$$

$$\begin{aligned}
C_{i,j} &\equiv -\left(\frac{\delta\alpha_j}{\Delta\theta}\right)^2 - \frac{\delta^2}{2r_j^2} - \left(\frac{\beta_i}{\Delta\eta}\right)^2, \\
D_i &\equiv \left(\frac{\beta_i}{\Delta\eta}\right)^2 - \frac{2\pi^2\beta_i^3 b \sin 2\pi\eta_i}{\Delta\eta}, \\
E_i &\equiv \left(\frac{\beta_i}{\Delta\eta}\right)^2 + \frac{2\pi^2\beta_i^3 b \sin 2\pi\eta_i}{\Delta\eta}.
\end{aligned} \tag{18}$$

The velocity components of the meridional flow are obtained from

$$u_{i,j} = \frac{\beta_i}{r_j} \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta\eta}, \quad w_{i,j} = -\frac{\alpha_j}{r_j} \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta\theta}. \tag{19}$$

The Poisson-type Equation (13) is approximated by the five-point formula which yields, for $\psi_{i,j}$,

$$\begin{aligned}
\psi_{i,j} &= \frac{1}{2} \left[\left(\frac{\delta\alpha_j}{\Delta\eta} \right)^2 + \left(\frac{\beta_i}{\Delta\eta} \right)^2 \right]^{-1} [D_i \psi_{i+1,j} + E_i \psi_{i-1,j} \\
&\quad + (A_j - \frac{\delta^2\alpha_j}{r_j\Delta\theta}) \psi_{i,j+1} + (B_j + \frac{\delta^2\alpha_j}{r_j\Delta\theta}) \psi_{i,j-1} - r_j \zeta_{i,j}].
\end{aligned} \tag{20}$$

This system of algebraic equations is solved with Gauss-Seidel line overrelaxation applied along lines of constant θ . The overrelaxation factor is 1.78. The iteration is halted after the k^{th} iteration if, at each grid point,

$$|\psi_{i,j}^k - \psi_{i,j}^{k-1}| < 10^{-4} |\psi_{i,j}^{k-1}|. \tag{21}$$

At the solid boundaries a one-sided first order difference equation is used to compute the vorticity ζ . Two such equations were tried,

the simple one which is used by most authors (at the bottom of the tank, for instance),

$$\zeta_{1,j} = \frac{2}{r_j} \left(\frac{\beta_1}{\Delta\eta} \right)^2 \psi_{2,j} \quad (22)$$

and another, which was found in a study involving curved boundaries¹² to be superior with regard to numerical stability,

$$\zeta_{1,j} = \frac{1}{4r_j} \left(\frac{\beta_1}{\Delta\eta} \right)^2 (\psi_{2,j} + 4\psi_{3,j} - \psi_{4,j}). \quad (23)$$

In this paper Equation (23) is used although it showed no advantage over Equation (22).

It can be seen from Equations (16) through (20) and Equation (23) that the computations at the inner points do not require knowledge of quantities at the singular point. (The subscript $i+1, j+1$ does not appear in these equations.)

The integration process is carried out in the following way: The vorticity $\zeta_{i,j}^{n+1}$ and subsequently the azimuthal velocity $v_{i,j}^{n+1}$ are computed at the inner points according to Equations (16) and (17). The calculation of $\psi_{i,j}^{n+1}$ follows with the aid of Equation (20). The cycle then concludes with the calculation of the surface vorticity.

The maximum stable time step, Δt_{\max} , beyond which numerical instability occurs, is determined by increasing the time step until oscillations from point to point in the ζ -values appear.

The accuracy of the computations is checked by using different space increments. This is easily done by varying the stretching parameters a and b defined in Equation (15). It is found for the two cases $Ro = 1$, $\delta = 1$, $Ek = 0.01$ and 0.001 that solutions obtained with different stretching values agree well except near the singular

point. The influence of this singularity on the solution is discussed in Section 4. Another way to change the space increments is to vary the number of grid points. For $Ro = 1$, $Ek = 0.001$ two grid systems, 41×41 and 51×51 , are used. The agreement is good.

4. RESULTS

As already noted, our main interest is focused on fluid motions in a rotating tank, that is flows with $Ro < \infty$. The examples selected for computation are compiled in Table 1. All cases are started from solid-body rotation at $t = 0$ and are continued to an almost steady state at t_{Final} , where t_{Final} is the earliest time at which

$$|\psi_{i,j}^n - \psi_{i,j}^{n-1}| \leq 10^{-4} |\psi_{i,j}^{n-1}| \quad (24)$$

is satisfied throughout the field. The computations were performed in double precision on an IBM 360-91 computer and in single precision on a CDC 6700 computer. Pictures of the flow field were made with a Stromberg-Carlson SC-4020 charactron plotter.

a. The Almost Steady Case

Two cases may be distinguished if the nonlinear inertial terms are negligible. For large Ekman numbers the pressure force is essentially balanced only by the friction force (slow motion). For $Ro \ll Ek \ll 1$ a balance is maintained among the Coriolis, friction, and pressure forces.

Slow-motion solutions have been obtained by Hort¹⁴ for $Ro = 1$, $\delta = 1$, $Ek = \infty$. Pao³ checked numerically for $Ro = 1$, $\delta = 1$ that the

TABLE 1
Compilation of the Calculated Examples

No.	Ro	Ek	δ	a	b	GRID
1	10^{-5}	0.01	1	0.2	0.1	41×41
2	10^{-5}	0.001	1	0.2	0.1	
3	10^{-4}	0.01	1	0.2	0.1	
4	1	0.01	1	0.0	0.1	
5	1	0.01	1	0.26	0.13	
6	1	0.005	1	0.15	0.1	
7	1	0.0025	1	0.2	0.1	
8	1	0.00125	1	0.2	0.1	
9	1	0.001	1	0.2	0.1	
10	1	0.001	1	0.26	0.13	
11	1	0.001	1	0.0	0.0	51×51
12	1	0.0002	1	0.2	0.1	51×51
13	1	0.01	3	0.0	0.1	41×41
14	1	0.001	3	0.2	0.1	
15	1	0.01	1/2	0.2	0.1	
16	1	0.01	1/3	0.2	0.1	
17	4	0.01	1	0.2	0.1	
18	4	0.004	1	0.2	0.1	
19	10	0.01	1	0.2	0.1	

slow-motion approximation is applicable for $0.125 < Ek \leq \infty$.

The linear theory predicts for the case $Ro \ll Ek \ll 1$ three distinct regions: The Ekman layers at the cover and at the bottom of the tank, the Stewartson layer at the side wall, and the geostrophic interior, for which the Taylor-Proudman theorem of inviscid fluids holds¹⁶. Numerical results verify this notion and reveal its limitation. Figure 2 shows lines of constant ψ , ζ , and v -values for $Ro = 10^{-4}$, $Ek = 0.01$, $\delta = 1$ and for $Ro = 10^{-5}$, $Ek = 0.001$, $\delta = 1$. The increments of the ψ , ζ , and v -values are recorded in Table 2. The case $Ro = 10^{-5}$, $Ek = 0.001$, in particular the lines of constant ζ , illustrates clearly the existence of the three regions predicted by the linear theory. Additional results obtained for $Ro = 10^{-5}$, $Ek = 0.01$, $\delta = 1$ agree so well with the case $Ro = 10^{-4}$, $Ek = 0.01$, $\delta = 1$ that the patterns are indistinguishable. The increase of the Ekman number from 0.001 to 0.01 shows that the (almost) inviscid interior has vanished. If we neglect the influence of the side wall, an analytic solution from the linear theory is available. This is the well-known Ekman solution. In Figure 3 the numerical values for v at the centerline $r = 0$ are compared with those of the Ekman solution. Again, the analytic values agree well with the numerical output for $Ek = 0.001$, whereas for $Ek = 0.01$ the numerical data reveal the influence of the side wall. A distinct asymmetry is displayed between cover and bottom for $Ek = 0.01$.

The Stewartson layer at the side wall is studied with a perturbation method⁹ and compared with the numerical output. Two different layers, one inside the other, must be distinguished: a layer of thickness $Ek^{1/4}$, and a layer of thickness $Ek^{1/3}$ inside the first layer and adjacent to the wall. The $Ek^{1/4}$ -layer is represented

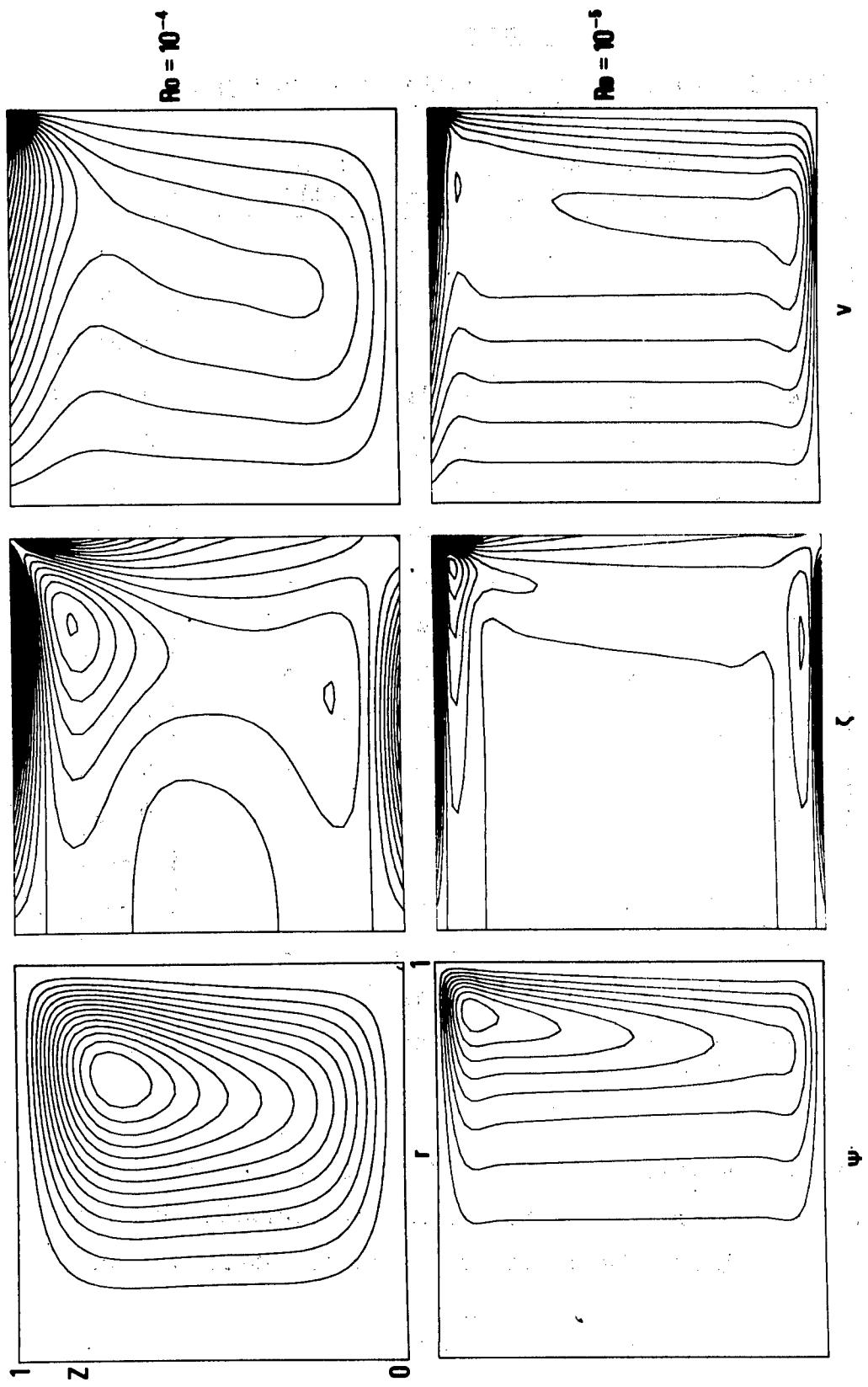


Figure 2 - Lines of constant ψ , ζ and v for $\delta = 1$, $Ro = 10^{-4}$, $Ek = 0.01$ and $Ro = 10^{-5}$, $Ek = 0.001$ at almost Steady State

TABLE 2
Data for the ψ , ζ , and v Patterns in the Figures

No.	Figure	Sign of ψ Near $r=1, z=1$	$\Delta\psi$	Sign of ζ on the Cover	$\Delta\zeta$	Δv
2	2	-	0.001	-	1.00	0.05
3	2	-	0.001	-	0.25	0.05
5	6	-	0.001	-	0.25	0.05
7	6	-	0.001	-	0.25	0.05
8	7	-	0.001	-	0.50	0.05
10	7	-	0.001	-	0.50	0.05
12	8	-	0.001	-	1.00	0.05
19	8	+	0.001	+	0.50	0.05
17	9	+	0.001	+	0.25	0.05
18	9	+	0.001	+	0.25	0.05
13	10	-	0.0001	-	0.25	0.05
16	10	-	0.001	-	0.25	0.05
14	11	-	0.001	-	0.50	0.05
2	15	-	0.001	-	--	0.05
10	19	-	0.001	-	--	0.05
12	20	-	0.001	-	--	--
12	21	-	0.001	-	--	0.05
19	22	+	0.001	+	0.50	0.05

The stream function ψ is specified to be zero at the boundaries. The vorticity ζ is zero on the centerline. The azimuthal velocity v is zero at all boundaries except at the cover, where $v = -r$.

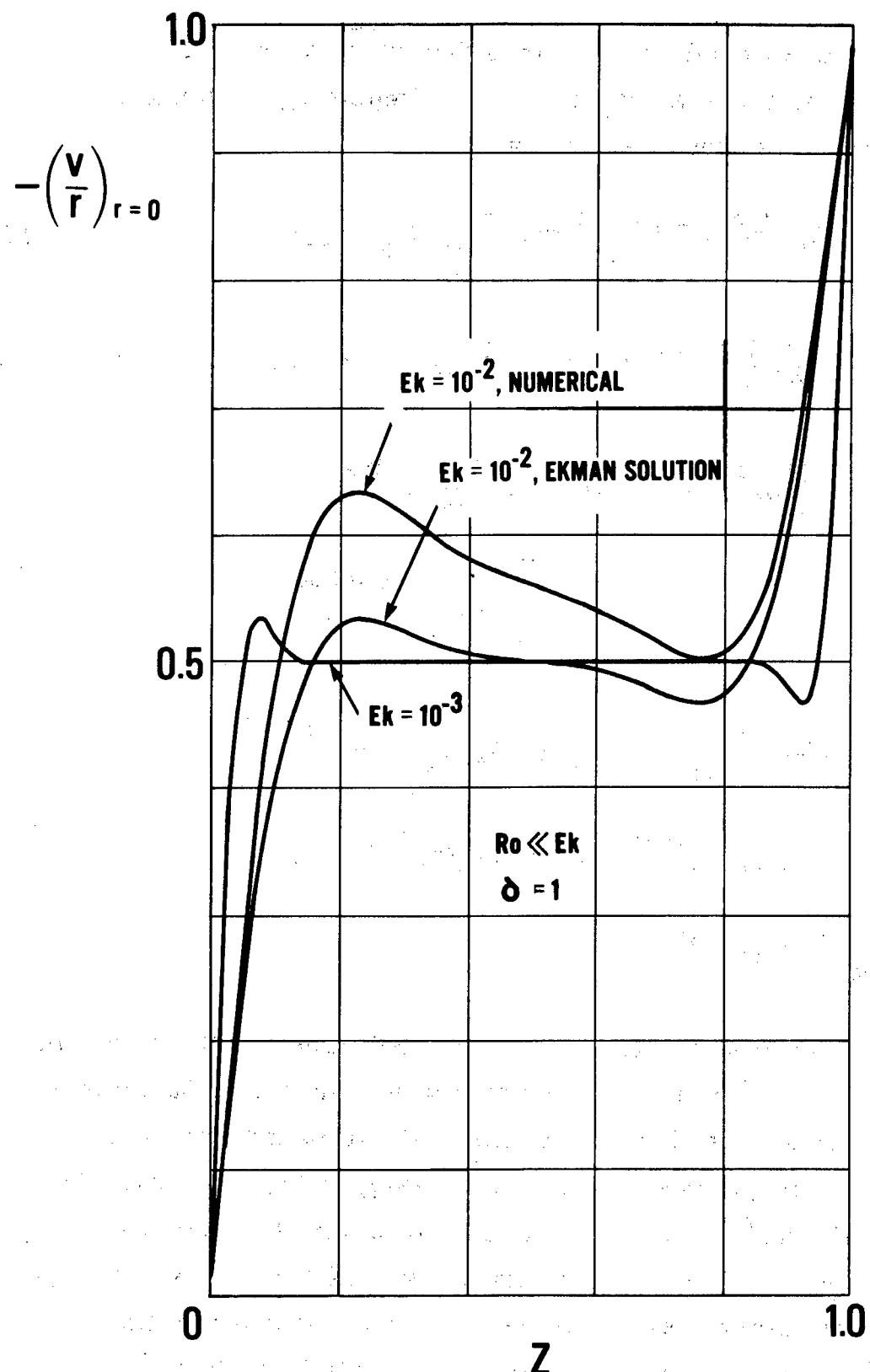


Figure 3 - Comparison of the Ekman Solution with the Numerical Results. $-(v/r)_{r=0}$ is plotted versus z .

by a closed-form solution, whereas the $Ek^{1/3}$ -layer must be expressed by a series expansion. For simplicity, the velocity components are presented for $\delta = 1$:

$$u = \frac{1}{2} Ek^{1/2} e^{-\sqrt{2}\sigma} - \frac{1}{2} Ek^{1/2} \sum_{k=1}^{\infty} (x_k)_{\tau\tau} \cos k\pi z, \quad (25)$$

$$v = -\frac{r}{2} + \frac{1}{2} e^{-\sqrt{2}\sigma} + Ek^{1/6} \sum_{k=1}^{\infty} x_k \cos k\pi z, \quad (26)$$

$$w = -\frac{1}{2} Ek^{1/2} - \frac{1}{\sqrt{2}} Ek^{1/4} (z - \frac{1}{2}) e^{-\sqrt{2}\sigma} + Ek^{1/6} \sum_{k=1}^{\infty} x_k \sin k\pi z, \quad (27)$$

where

$$x_k = (-1)^k \frac{2}{\sqrt{3} k\pi} \gamma_k \sin \left(\frac{\sqrt{3}}{2} \gamma_k \tau \right) e^{-\gamma_k/2 \tau},$$

$$\sigma = (1-r)/Ek^{1/4}, \quad \tau = (1-r)/Ek^{1/3},$$

$$\gamma_k = (2k\pi)^{1/3}. \quad (28)$$

The first term in v and w is the geostrophic mode of the interior. In Figures 4 and 5 the v - and w -components are plotted against r . The numerical results are compared with the analytic data for a) the geostrophic mode and the $Ek^{1/4}$ -law, and b) the geostrophic mode, the $Ek^{1/4}$ -law, and the first two nonzero terms of the $Ek^{1/3}$ -series. For the v -component both analytic curves are in good agreement with the numerical data. For the w -component the $Ek^{1/4}$ -law alone is insufficient to describe the Stewartson layer. In

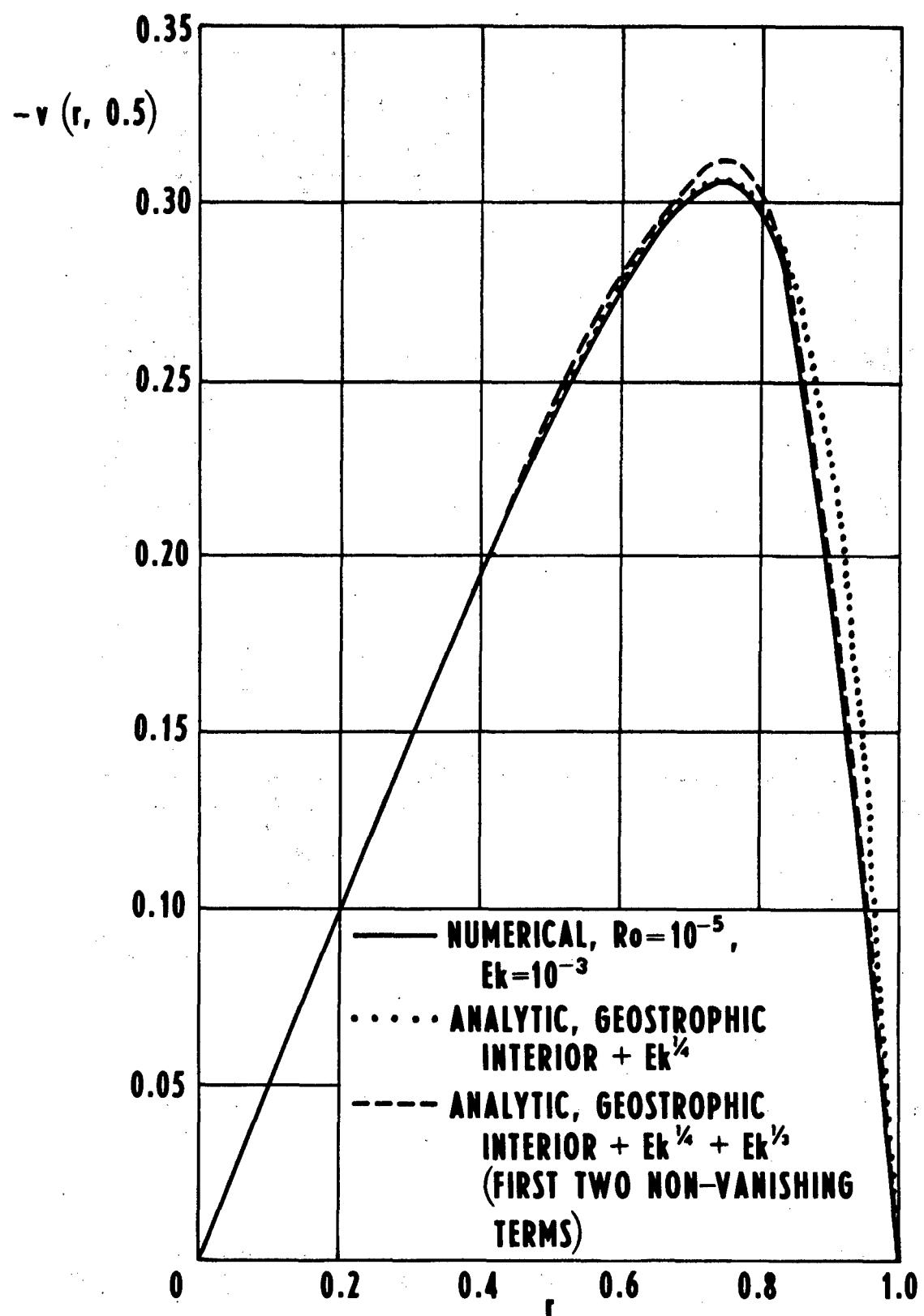


Figure 4 - Analytic and Numerical Results for the Stewartson Layer. $-v$ is Plotted Versus r at $z = 1/2$

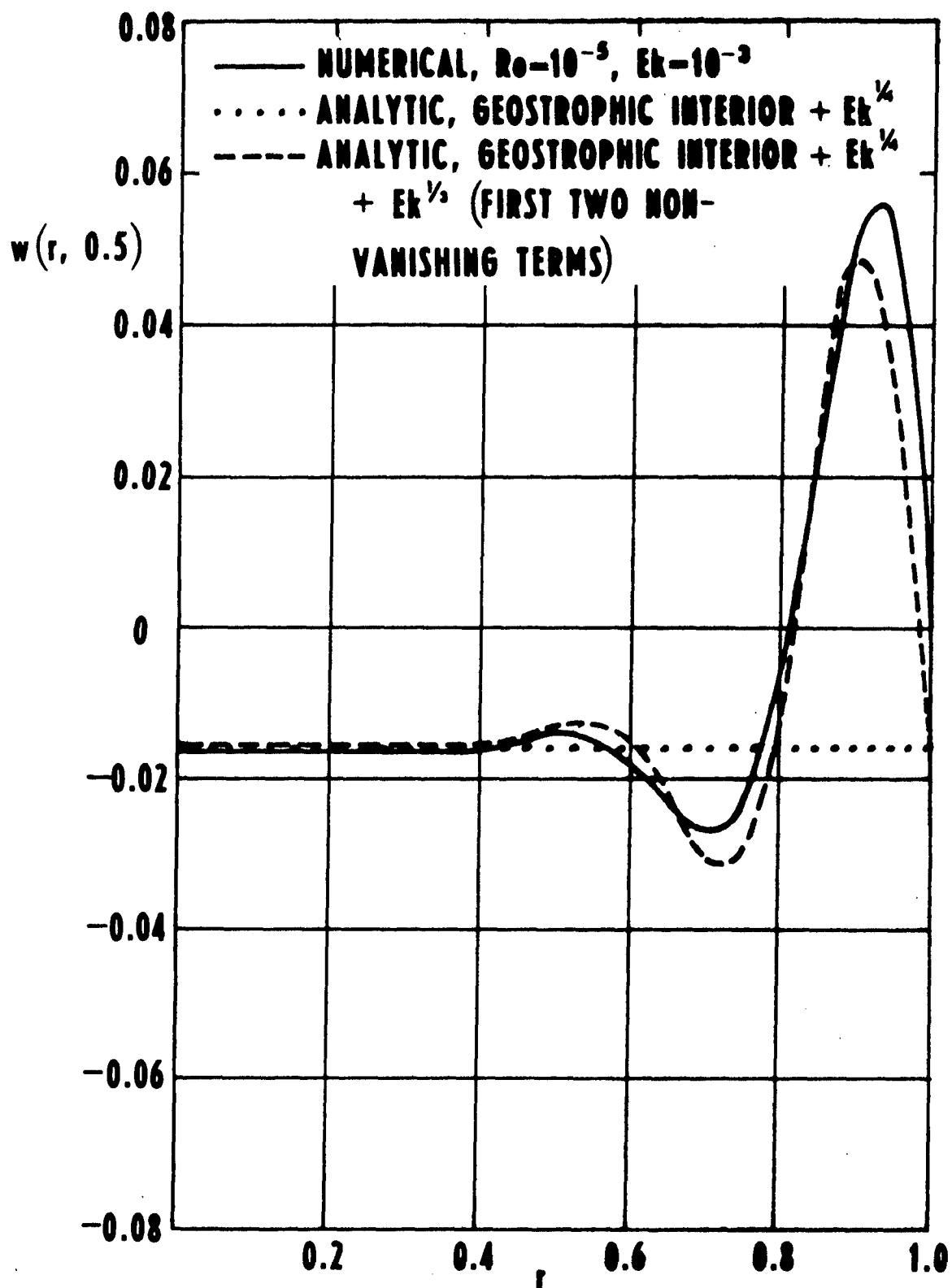


Figure 5 - Analytic and Numerical Results for the Stewartson Layer. w is Plotted Versus r at $z = 1/2$

fact, at $z = \frac{1}{2}$, the contribution of the $Ek^{1/4}$ -law vanishes.

The inclusion of the nonlinear terms into the equations of motion modifies the flow characteristics. Figures 6 through 8 show flow patterns for $Ro = 1$, $\delta = 1$, and various Ek . The output for $Ek = 0.01$ can be compared with Pao's computation³. The overall streamline patterns agree well with each other, although deviations occur near the singular point. This problem is discussed in more detail below. In Figures 8 and 9 flow patterns are displayed for $Ro = 10$ and 4, $\delta = 1$, and two different Ekman numbers. Cell-type motions occur which are expected around $Ro = 4$ according to similarity solutions¹⁷. Results in Figures 10 through 12 exhibit the effect of various δ . In the limit $\delta \rightarrow 0$ the solutions approach the similarity solutions obtained by Lance and Rogers¹⁵. This is demonstrated in Figure 12 by a graph of $-v/r$ versus z at $r = 0$.

The results for different Ro , Ek , δ have one common feature. Below a certain value of the Ekman number, spatial undulations of the streamlines occur indicating the transition from an elliptic-type to a hyperbolic-type solution. This distinction is most apparent in the two linear cases discussed at the beginning of this section. The slow-motion solution is clearly elliptic, whereas the Ekman solution and the geostrophic mode are parabolic (as an asymptotic limit of the hyperbolic time-dependent flow). For $Ro = 1$, $\delta = 1$ the critical Ekman number, at which the transition occurs, is 0.005. This value decreases for $Ro > 1$ and increases for $Ro < 1$. Variation of δ appears to have only a minor effect on the value of the critical Ekman number.

In Figure 13 a pathline is displayed for $Ro = 1$, $Ek = 0.0002$, $\delta = 1$ in the rotating frame. The starting point is arbitrarily chosen at $r = 0.489$, $z = 0.959$. The fluid particle follows a trajectory which

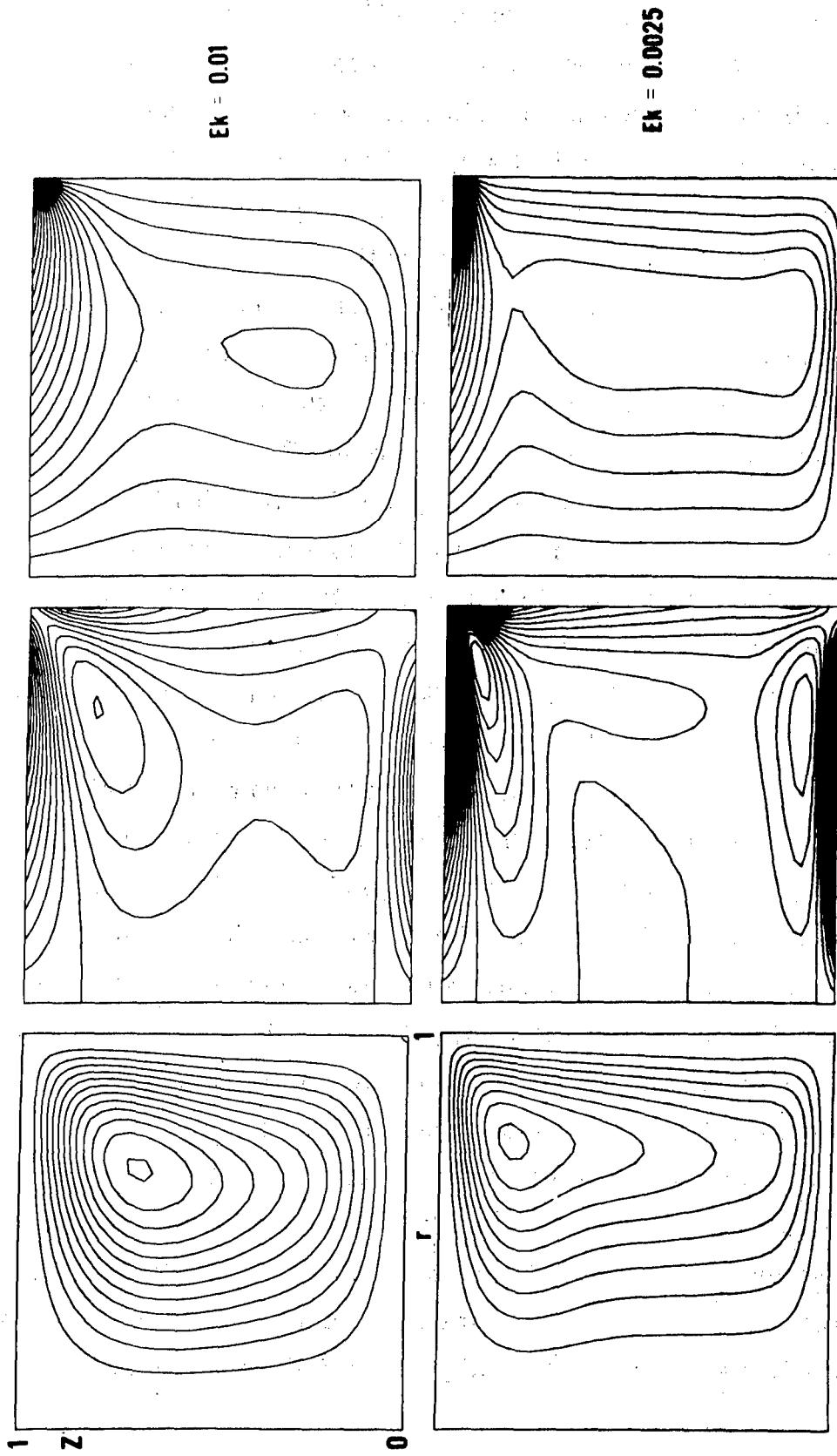


Figure 6 - Lines of Constant ψ , ζ , and v for $Ro = 1$, $\delta = 1$, $Ek = 0.01$ and $Ek = 0.0025$ at almost Steady State

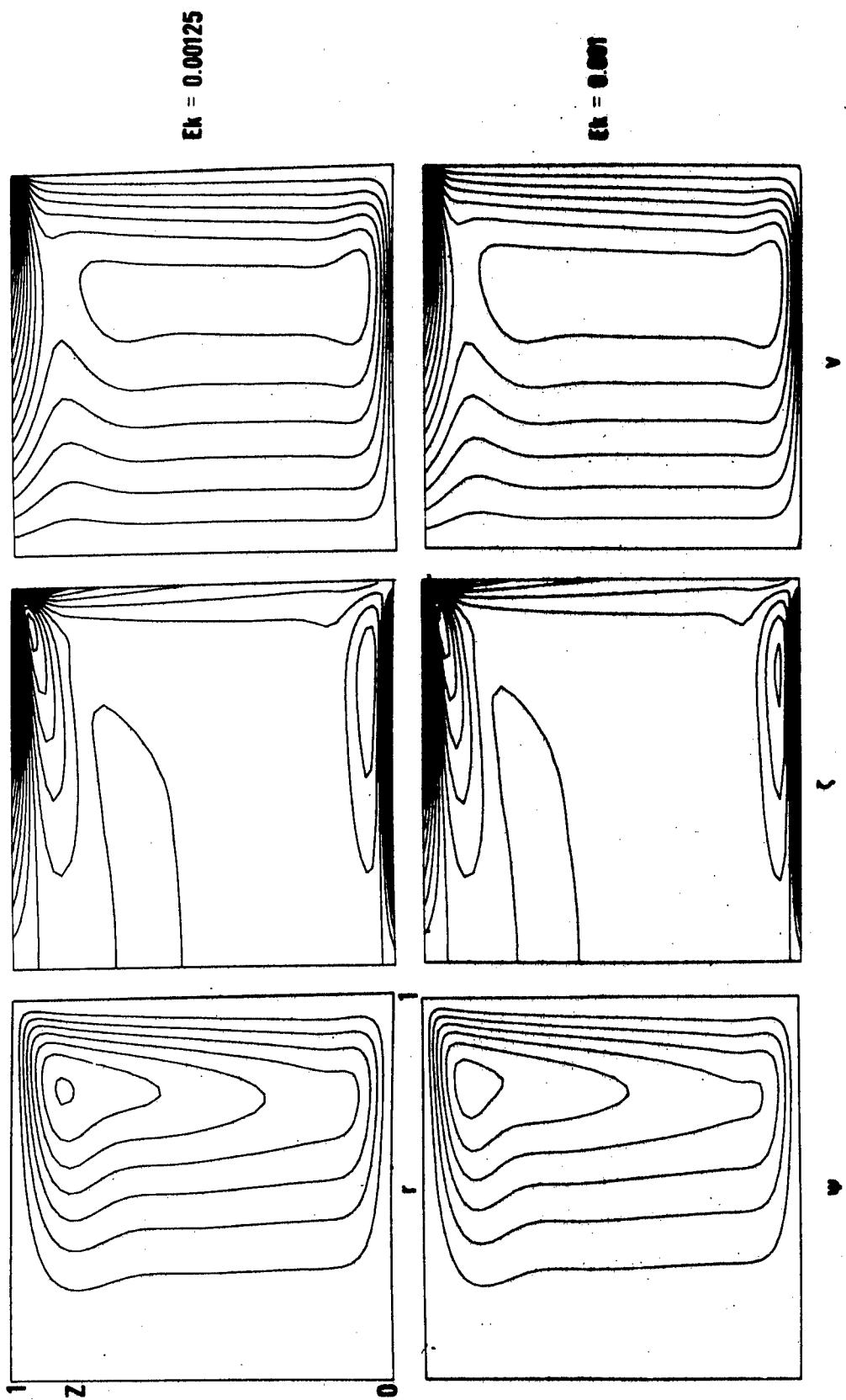


Figure 7 - Lines of Constant ψ , ξ , and v for $Ro = 1$, $\delta = 1$, $Ek = 0.00125$ and $Ek = 0.001$ at almost Steady State

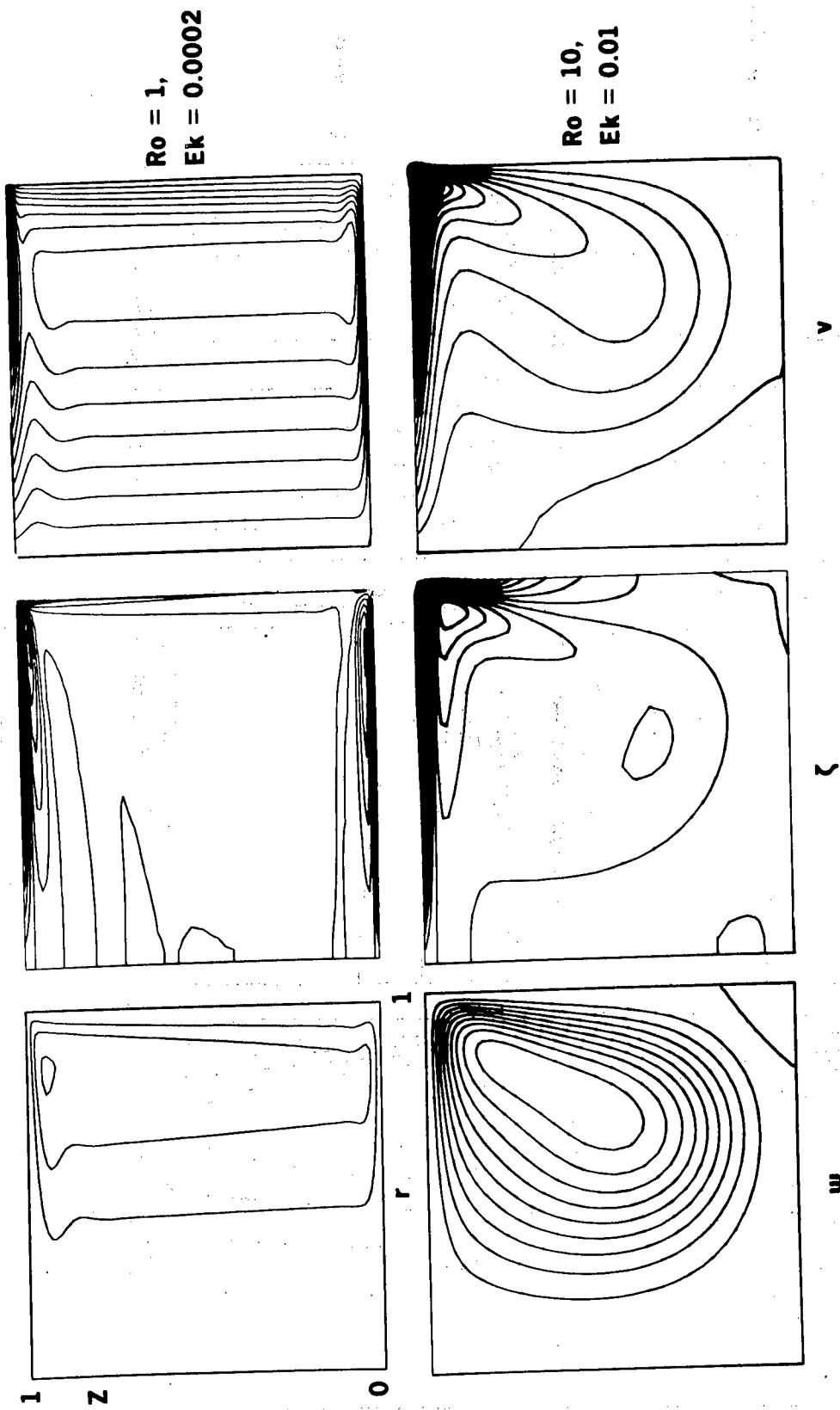


Figure 8 - Lines of Constant ψ , ζ , and v for $Ro = 1, \delta = 1, EK = 0.0002$ and $Ro = 10, \delta = 1, EK = 0.01$ at almost Steady State

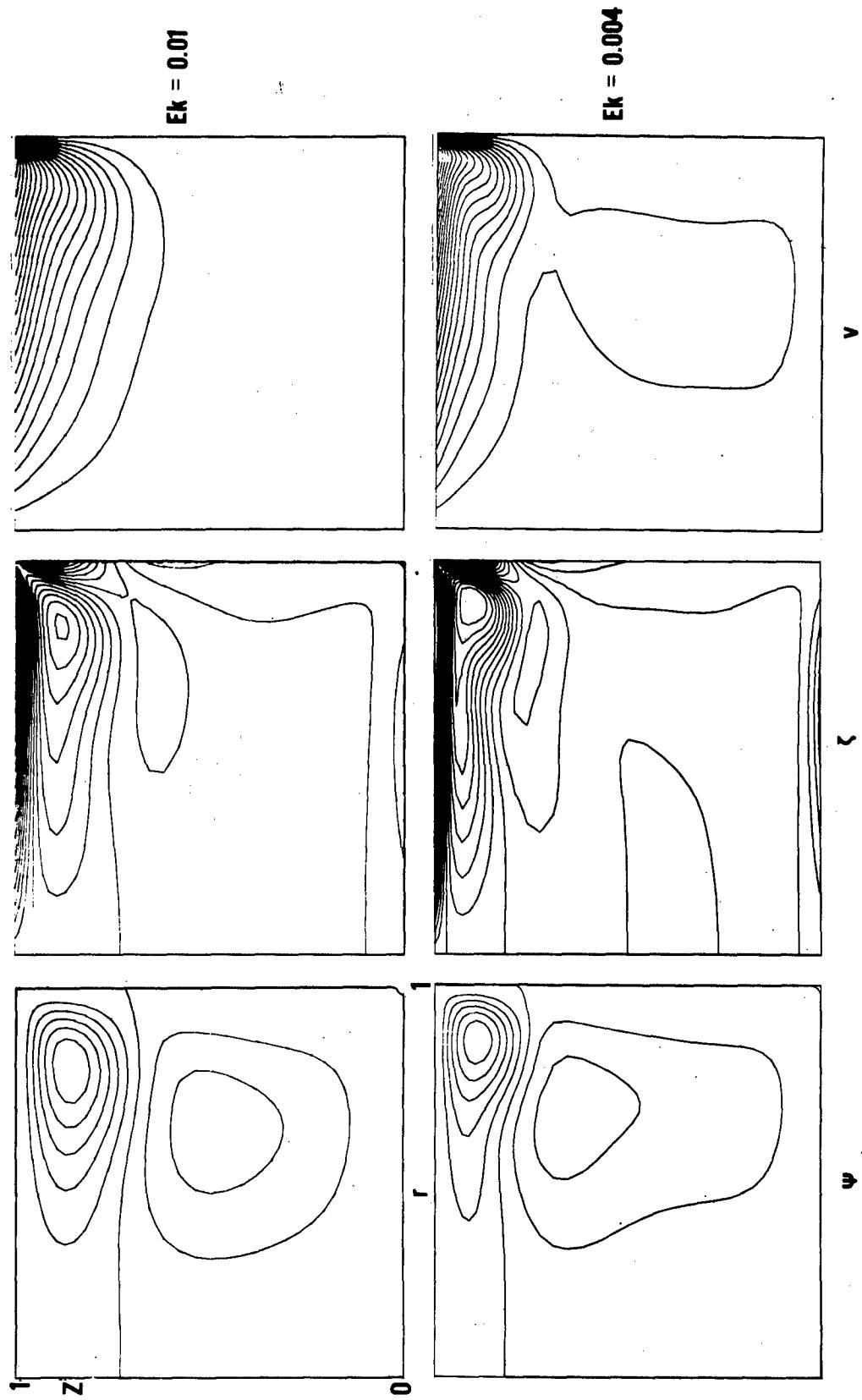


Figure 9 - Lines of Constant ψ , ζ , and v for $Ro = 4$, $\delta = 1$, $Ek = 0.01$ and $Ek = 0.004$ at almost Steady State

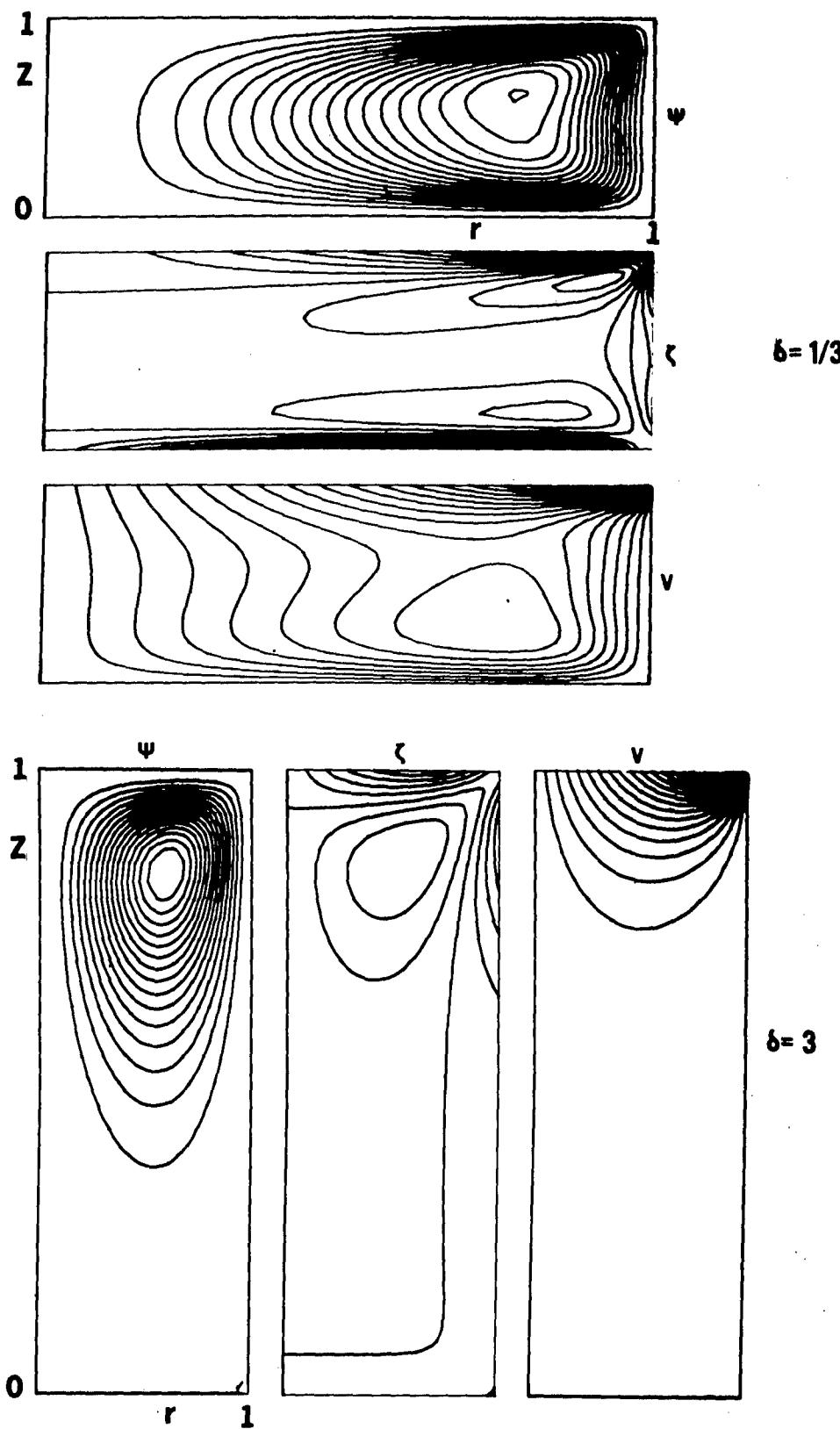


Figure 10 - Lines of Constant ψ , ζ , and v for $Ro = 1$,
 $Ek = 0.01$, $\delta = 3$ and $\delta = 1/3$ at almost Steady State

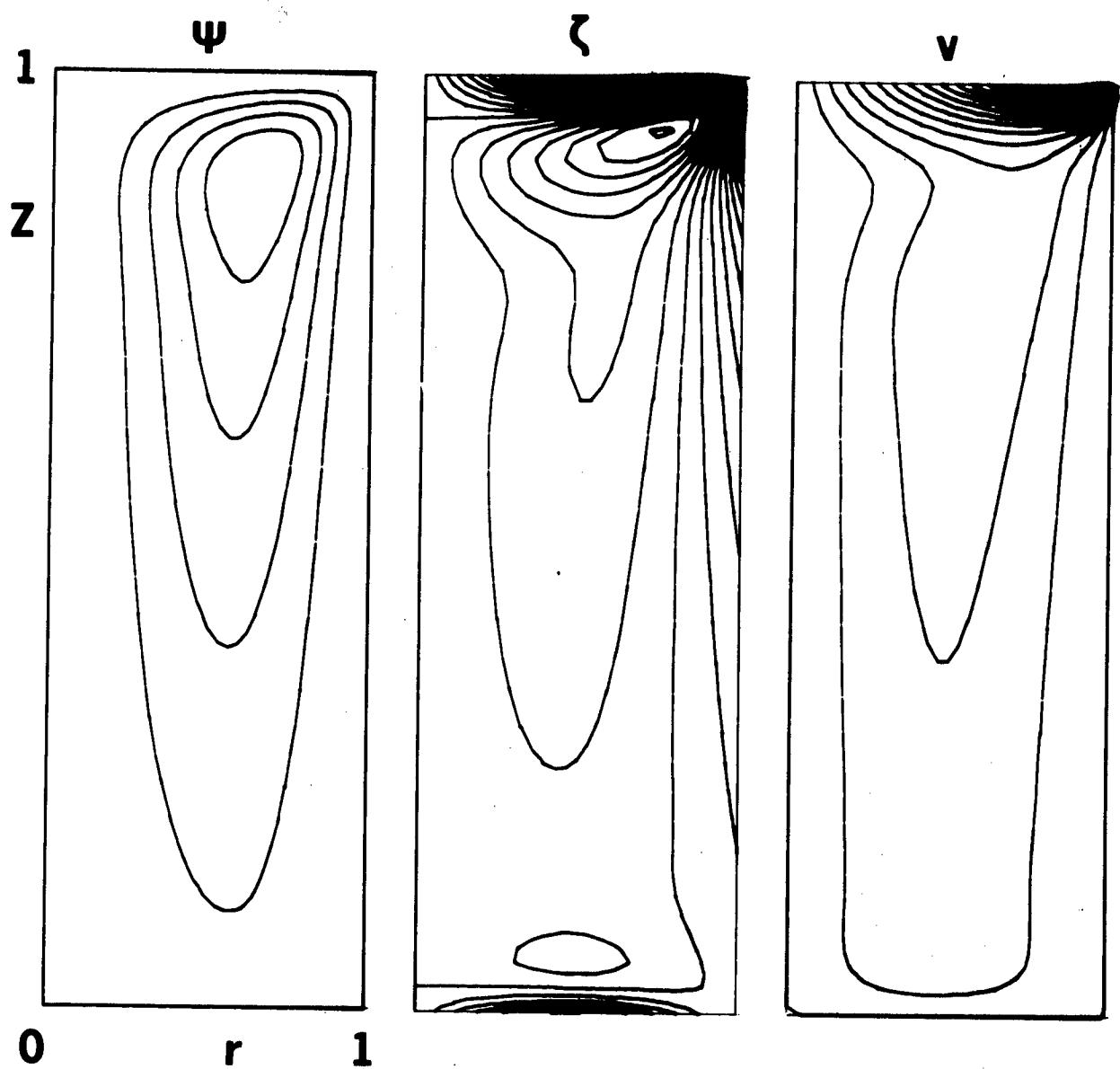


Figure 11 - Lines of Constant ψ , ζ , and v for $Ro = 1$, $Ek = 0.001$, $\delta = 3$
at almost Steady State

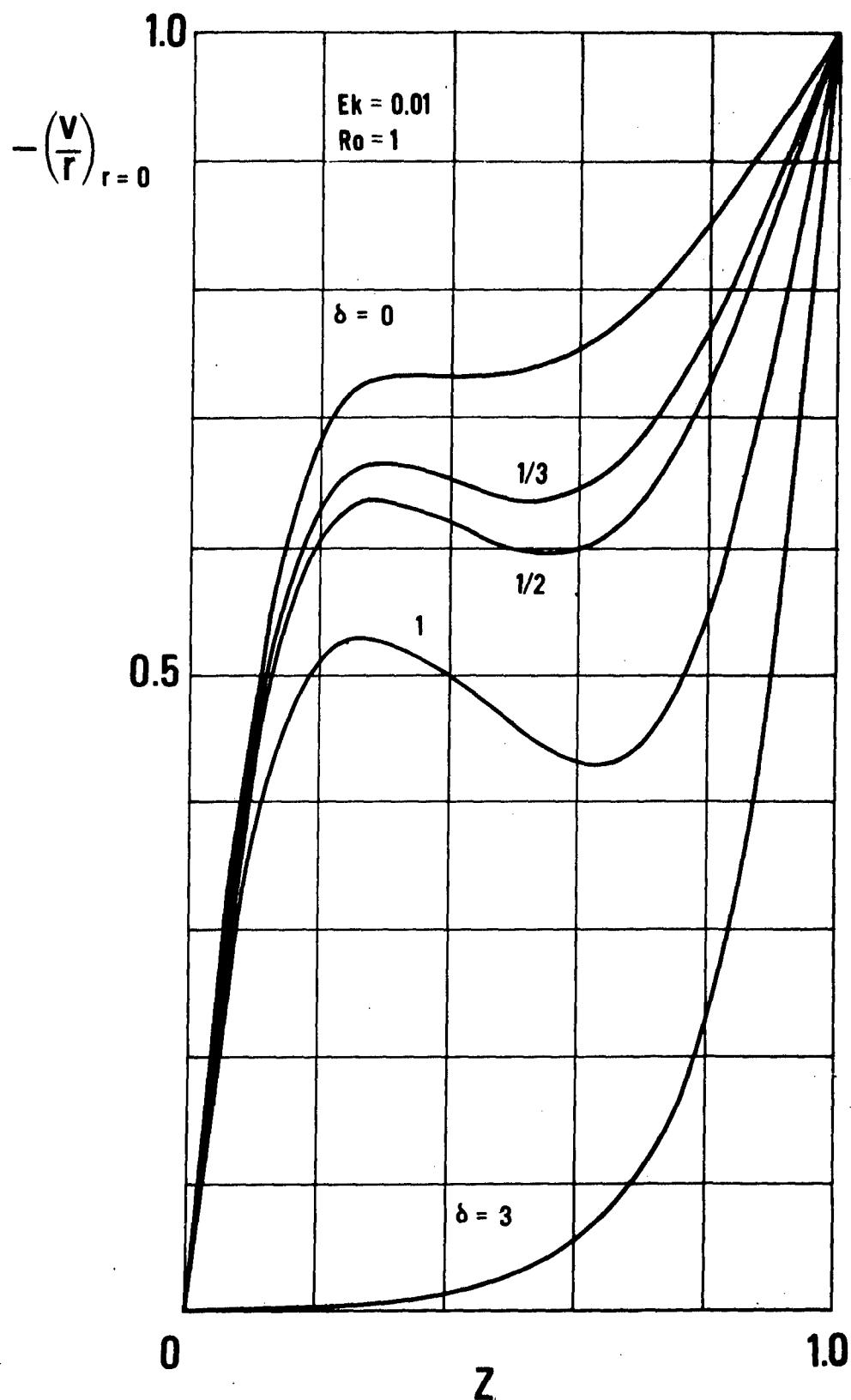


Figure 12 - $-(v/r)_{r=0}$ as a Function of z for Various δ

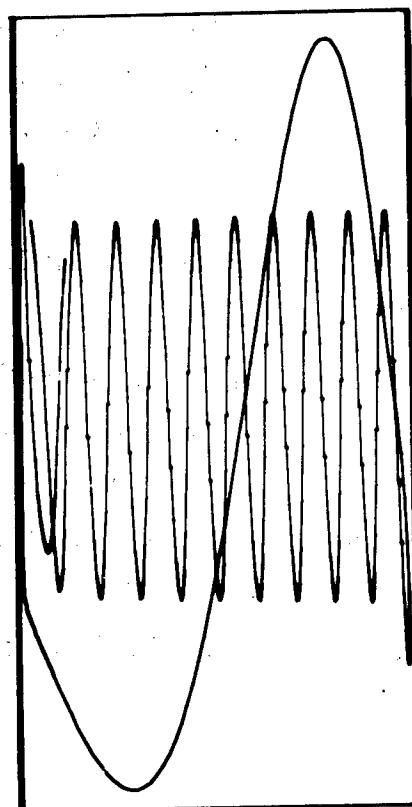
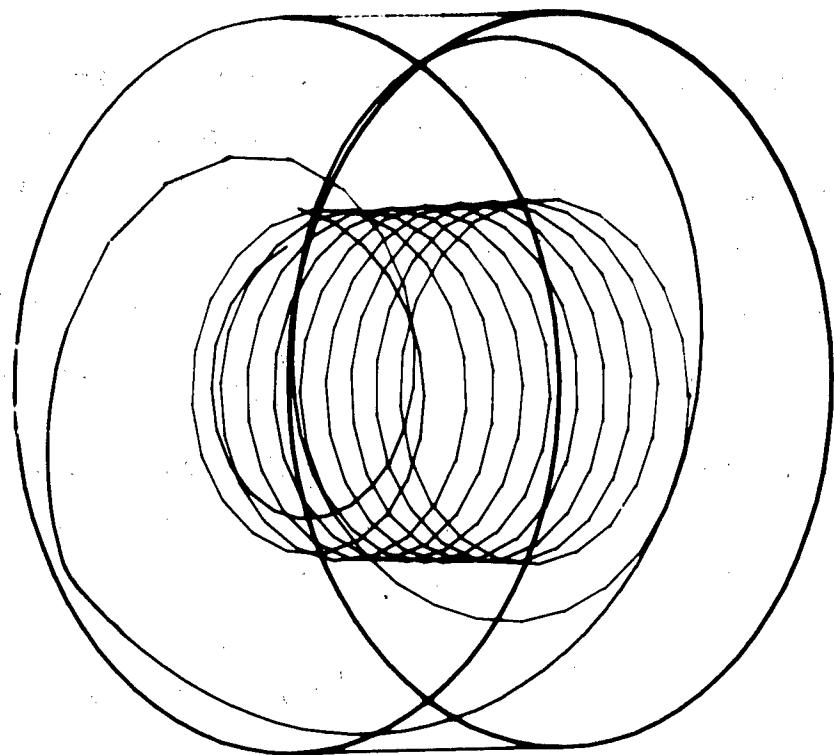


Figure 13 - Pathline for $Ro = 1$, $Ek = 0.0002$, $\delta = 1$. Perspective View at almost Steady State

consists of a downward spiral with almost no radial variation and an upward spiral near the side wall with only one revolution. The time for the particle to return to the vicinity of its initial position is about 22 revolutions of the tank ($t = 140$).

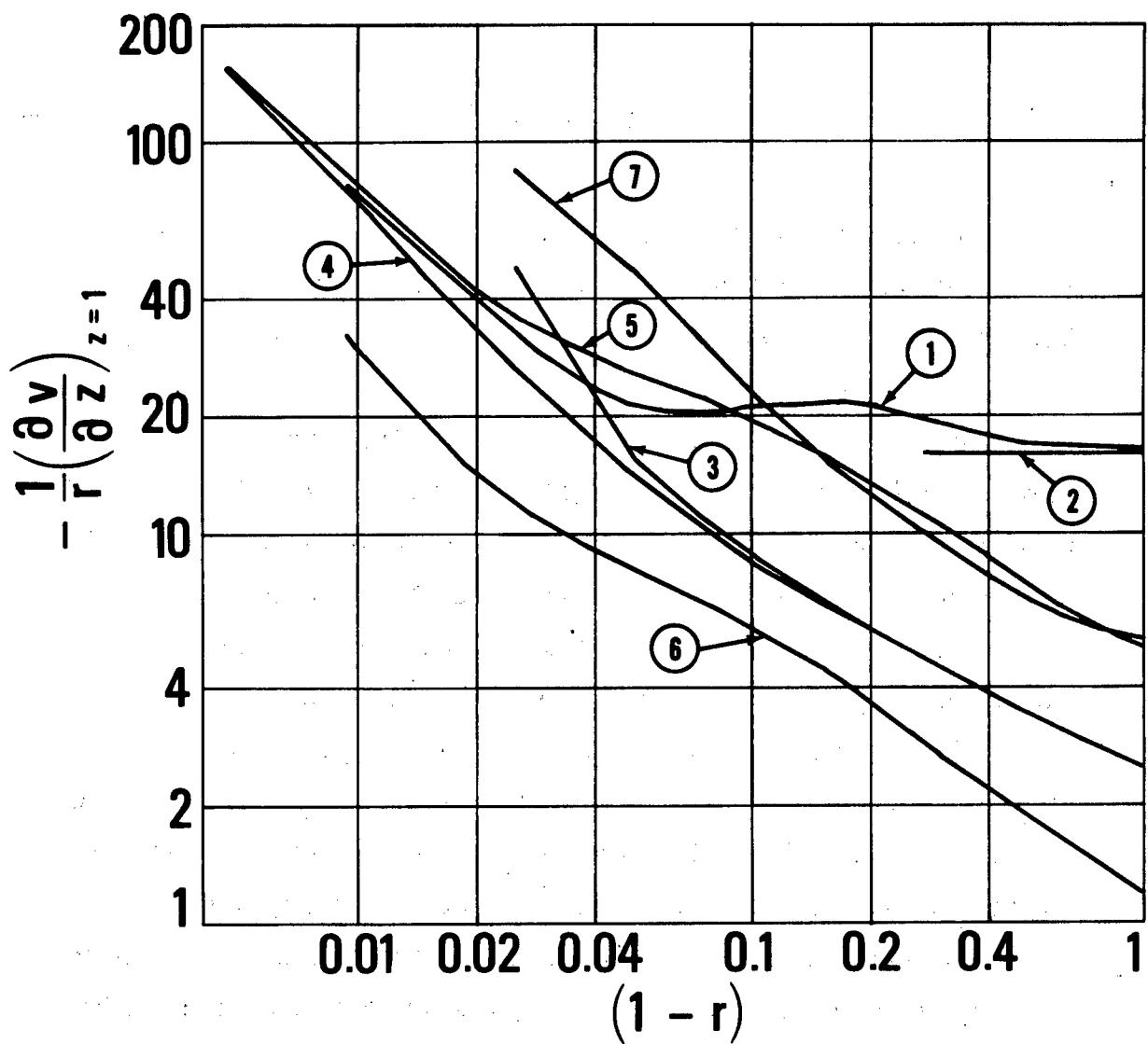
The singularity of the flow at $r = 1$, $z = 1$ requires special attention. In Figure 14 the function $-\frac{1}{r} \left(\frac{\partial v}{\partial z} \right)_{z=1}$ is plotted against $(1-r)$ on a double-logarithmic scale for various values of Ro , Ek , and δ . The dimensional quantity $\partial v' / \partial z'$ becomes unbounded at the singular point ($r' = L$, $z' = H$) according to

$$\lim_{r' \rightarrow L} -\left(\frac{\partial v'}{\partial z'} \right)_{z'=H} = c \frac{L\omega}{L-r'}, \quad c \approx 0.73, \quad (29)$$

where c does not depend on Ro , Ek , or δ . Hence, the singularity is a local effect which can be considered in a numerical scheme by building in a local series expansion. On the other hand, the torques exerted on the cover and the wall are always logarithmically singular. Figure 14 reveals that for $Ro = 1$, $Ek = 0.01$ the grid system with $a = 0$, $b = 0.1$ gives a poor representation of the near-corner region.

b. The Transient Case

Unsteady rotating flows exhibit parabolic or hyperbolic properties in time. The slow-motion solution¹⁴, for example, is a pure diffusion process and is, thus, parabolic. The linear theory of inviscid flow, in which the pressure gradient, the Coriolis force, and the local acceleration balance each other, reveals flows of hyperbolic nature⁹. This can be seen immediately for the vorticity equation cited in the introduction. If $Ek = 0$, an infinite but countable number of modes can be obtained. Their frequencies are



- ① $Ro = 10^{-5}$, $Ek = 10^{-3}$, $\delta = 1$
- ② EKMAN SOLUTION
- ③ $Ro = 1$, $Ek = 0.01$, $\delta = 1$, $a = 0$, $b = 0.1$
- ④ $Ro = 1$, $Ek = 0.01$, $\delta = 1$, $a = 0.26$, $b = 0.13$
- ⑤ $Ro = 1$, $Ek = 10^{-3}$, $\delta = 1$
- ⑥ $Ro = 1$, $Ek = 0.01$, $\delta = 1/3$
- ⑦ $Ro = 1$, $Ek = 0.01$, $\delta = 3$

Figure 14 - $\frac{1}{r}(\partial v / \partial z)_{z=1}$ as a Function of $1-r$ for Various Ro , Ek , and δ

$$f = \frac{2m}{\sqrt{\ell^2 + m^2}} , \quad (30)$$

where ℓ and m are the eigenvalues for the r - and z -components, respectively. The waves connected with these modes are called "inertial waves"⁹. The zero mode, which is time-independent, represents the geostrophic motion.

Modifications of the inviscid flows occur in the Ekman and Stewartson layers in order to satisfy the boundary condition of no-slip. In Figure 15 a sequence of ψ - and v -patterns for $Ro = 10^{-5}$, $Ek = 0.001$, $\delta = 1$ over the transient period is presented. A more detailed time history for the same case is recorded in Figure 16, where $-(\zeta/r)_{r=0}$ at the cover and at the bottom is plotted against time. After the sudden change of the cover's angular speed, a boundary layer on the cover develops whose $-(\zeta/r)_{r=0}$ -value reaches a peak at $t = 1.5$. This time agrees with the spin-up theory which estimates $t \approx 1$ for the initial phase⁹. Afterwards, inertial oscillations are visible. They appear in the streamline patterns in the form of temporal oscillations and produce cell-type motions in the center of the tank (Figure 15, $t = 3.98, 9.98$). With increasing time the inertial oscillations are damped, the local cell vanishes, and the flow reaches a steady state. Both curves in Figure 16 approach asymptotically the value $\frac{1}{2} Ek^{-1/2} = 15.811$ of the Ekman-layer solution.

With increasing Ekman number the influence of viscosity is felt in the interior. In Figure 17 the function $-(\zeta/r)_{r=0}$ at the cover and at the bottom is plotted against time for $Ro = 10^{-5}$, $Ek = 0.01$, $\delta = 1$. Inertial oscillations have almost vanished. In

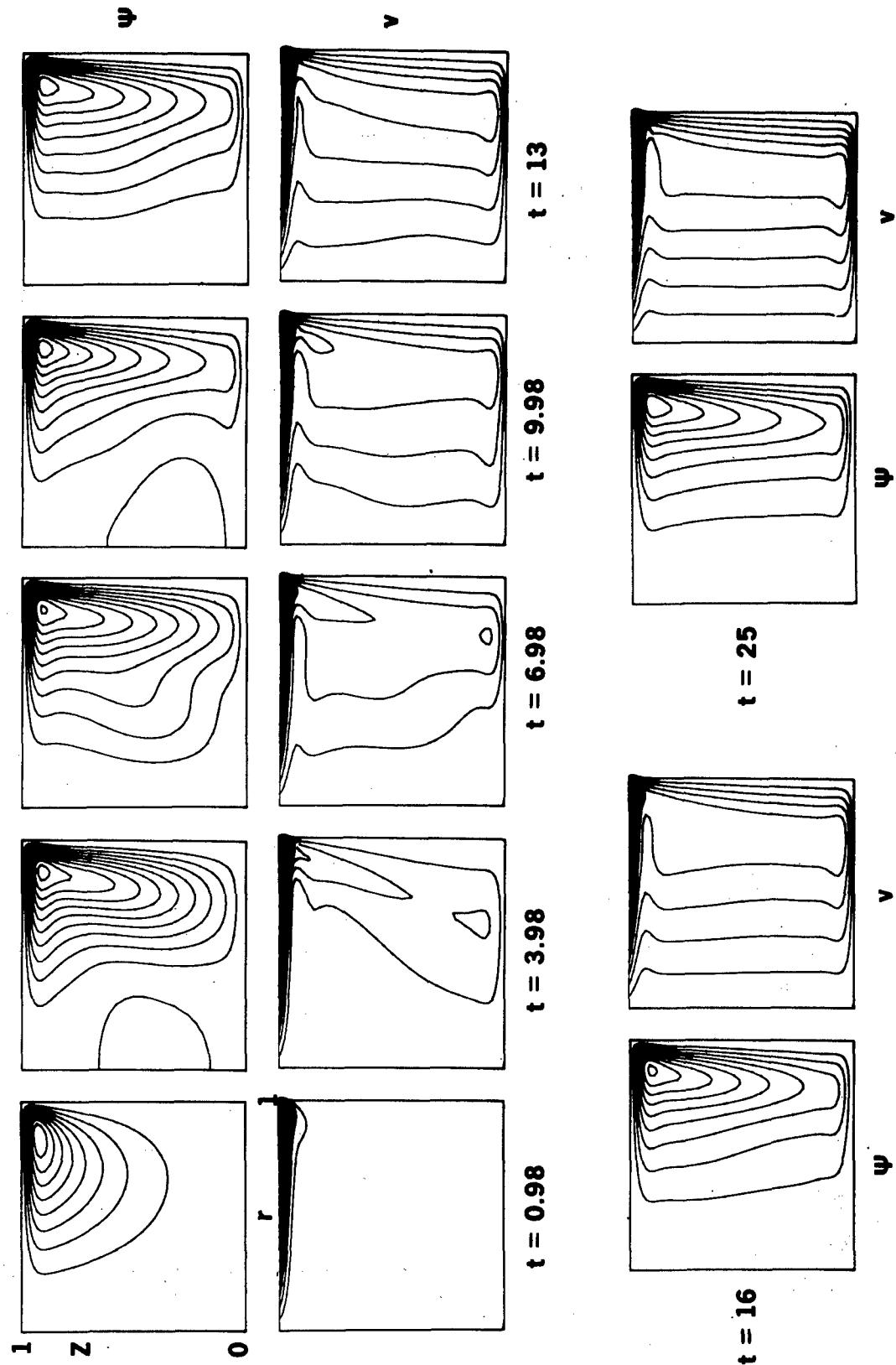


Figure 15 - Lines of Constant ψ and v for $Ro = 10^{-5}$, $Ek = 0.001$, $\delta = 1$ at Various t . The almost Steady State at $t_{FINAL} = 103$ is included in Figure 2

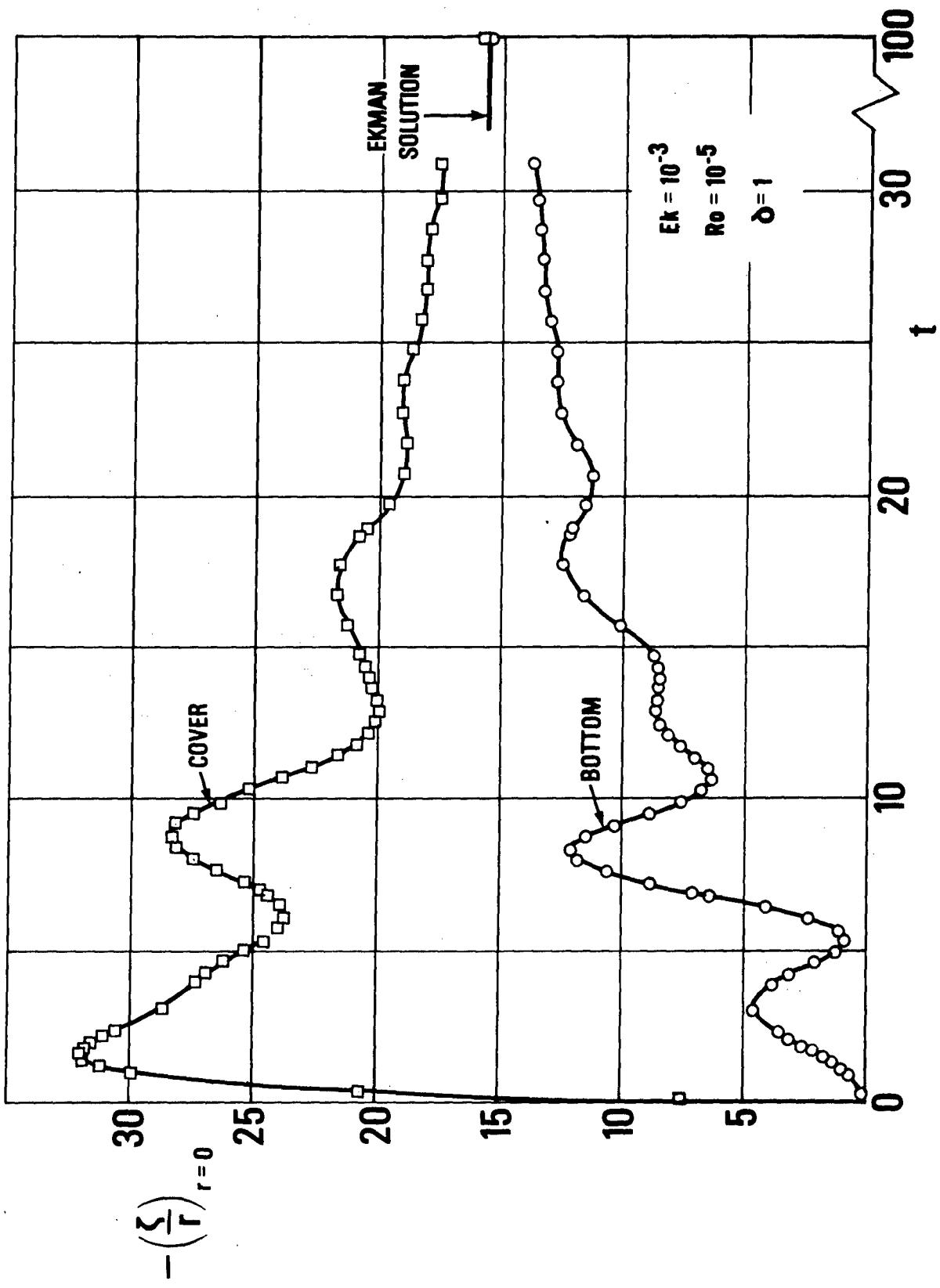


Figure 16 - $-(\xi/r)_{r=0}$ at Bottom and Cover as a Function of t for $Ro = 10^{-5}$, $Ek = 0.001$, $\delta = 1$

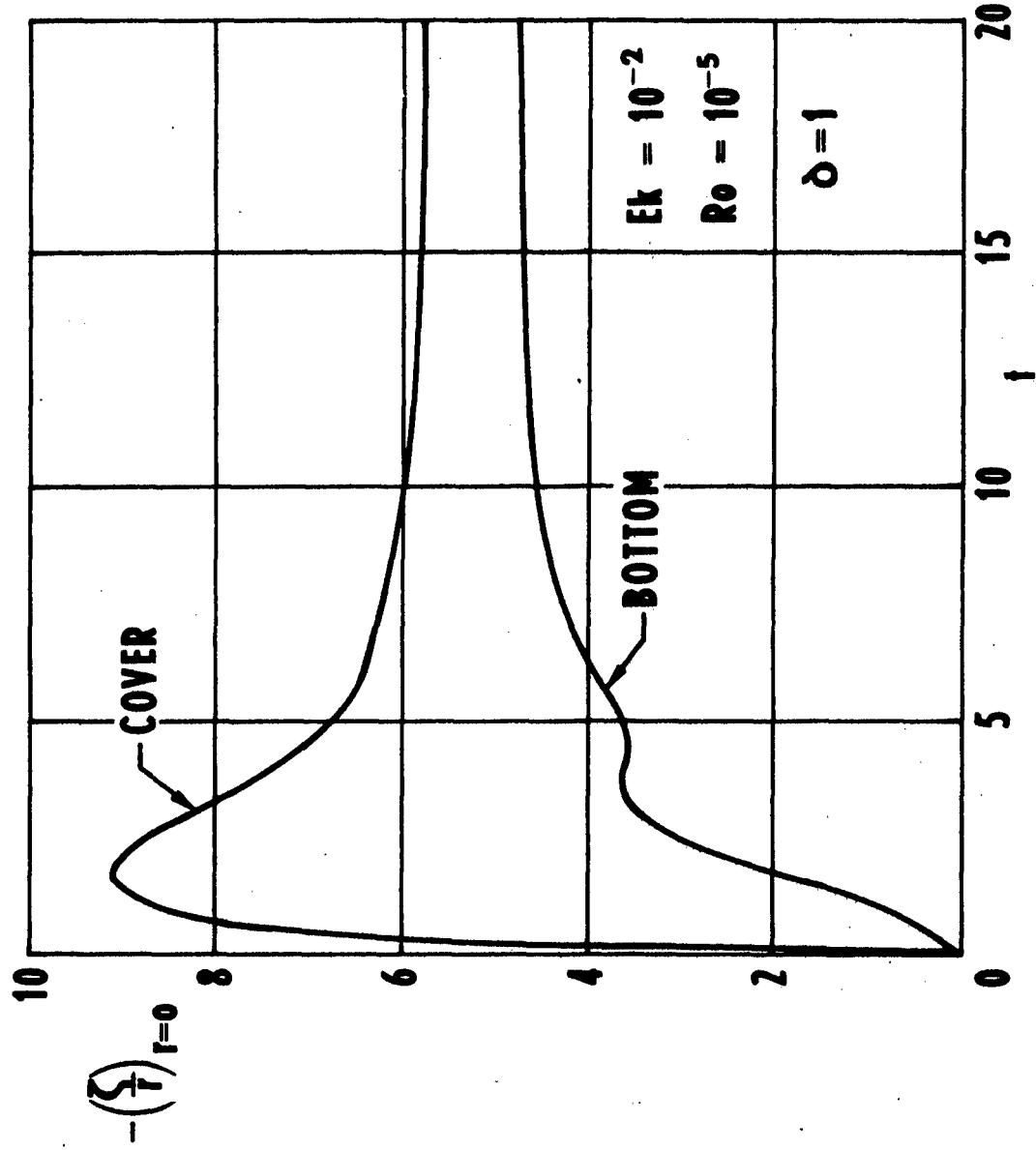


Figure 17 - $-(\zeta/r)|_{r=0}$ at Bottom and Cover as a Function of t for $Ro = 10^{-5}$, $Ek = 0.01$, $\delta = 1$

Figure 18 the flow behavior away from the boundary layers is displayed. The function $-(v/r)_{r=0}$ at $z = \frac{1}{2}$ is plotted against time for both $Ek = 0.001$ and $Ek = 0.01$ ($Ro = 10^{-5}$, $\delta = 1$). Inertial oscillations are observed only for the flow with the smaller Ekman number. No single mode can be identified. This indicates the existence of nonzero eigenvalues λ in Equation (30).

As long as the Ekman number is sufficiently small, that is, $Ek \ll 0.01$, inertial oscillations also occur if nonlinear effects are present. For $Ro = 1$, $Ek = 0.001$, $\delta = 1$ a time sequence of ψ - and v -patterns is shown in Figure 19. Again, a cell is visible in the center of the tank as in the linear case $Ro = 10^{-5}$, $Ek = 0.001$. A new phenomenon in Figure 19 is the appearance of a local region of positive v . Pao³ computed the case $Ro = 1$, $Ek = 0.001$ up to $t = 4.2$. His streamlines agree well with ours, but his picture does not show the local cell at $t = 4.2$.

For $Ro = 1$, $Ek = 0.001$, $\delta = 1$ a computer-generated movie has been made which shows the transient stage for ψ and v . The movie clearly reveals the time oscillations which cannot easily be detected in Figure 19.

Higher order modes are observed for decreasing Ekman number. In Figures 20 and 21 the ψ - and v -patterns are displayed for the transient period of the case $Ro = 1$, $Ek = 0.0002$, $\delta = 1$. At $t = 4.50$ and 11.25 two cells at the centerline are visible. The whole sequence shows three distinct time periods when local cells are present. It may be mentioned that Pao³ computed this case up to $t = 3.0$. His strong undulations of the streamlines near the singular point are not verified by our calculations.

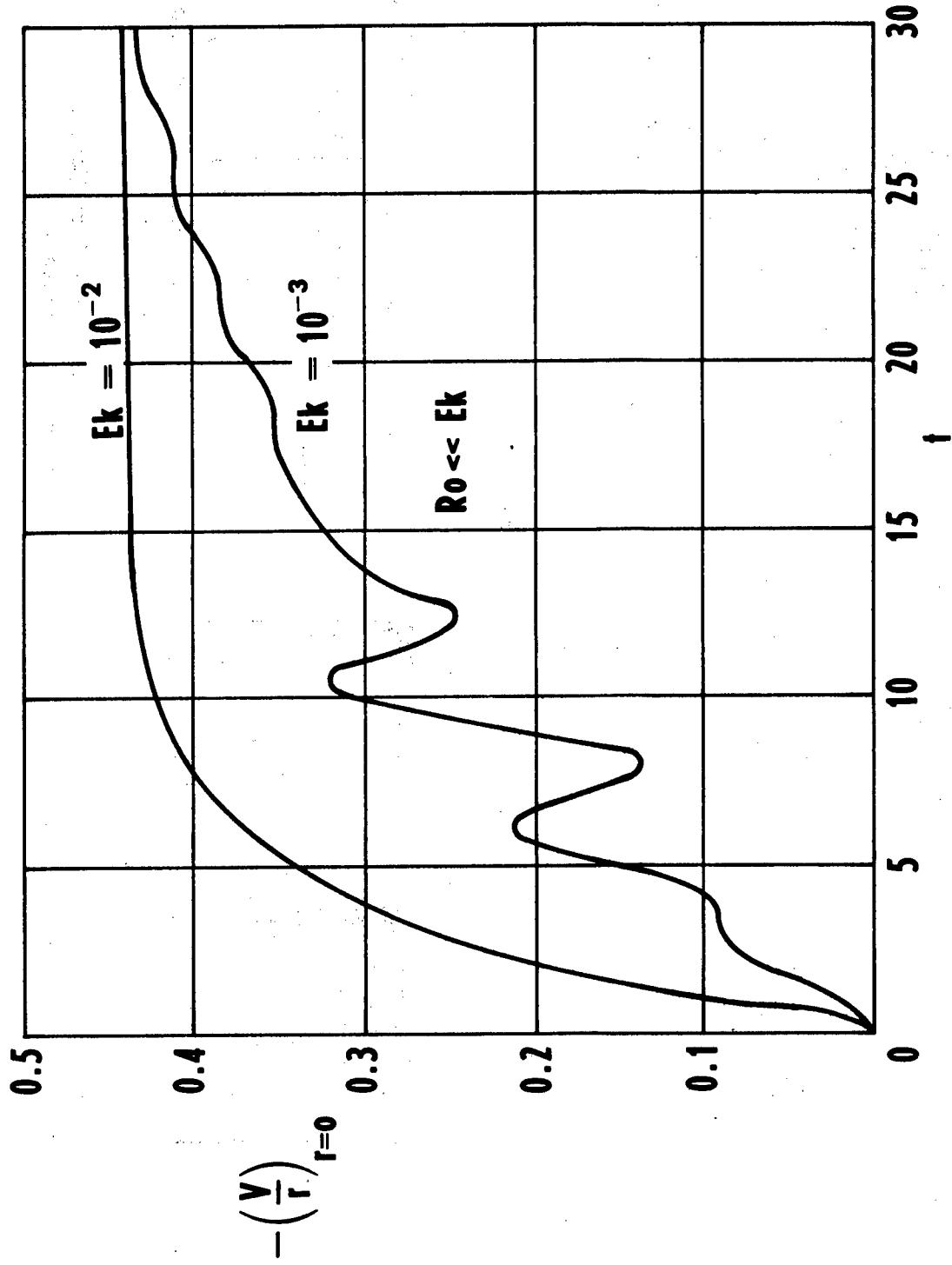


Figure 18 - $-(v/r)_{r=0}$ at $z = 1/2$ as a Function of t for $Ro = 10^{-5}$, $\delta = 1$, $Ek = 0.001$ and 0.01

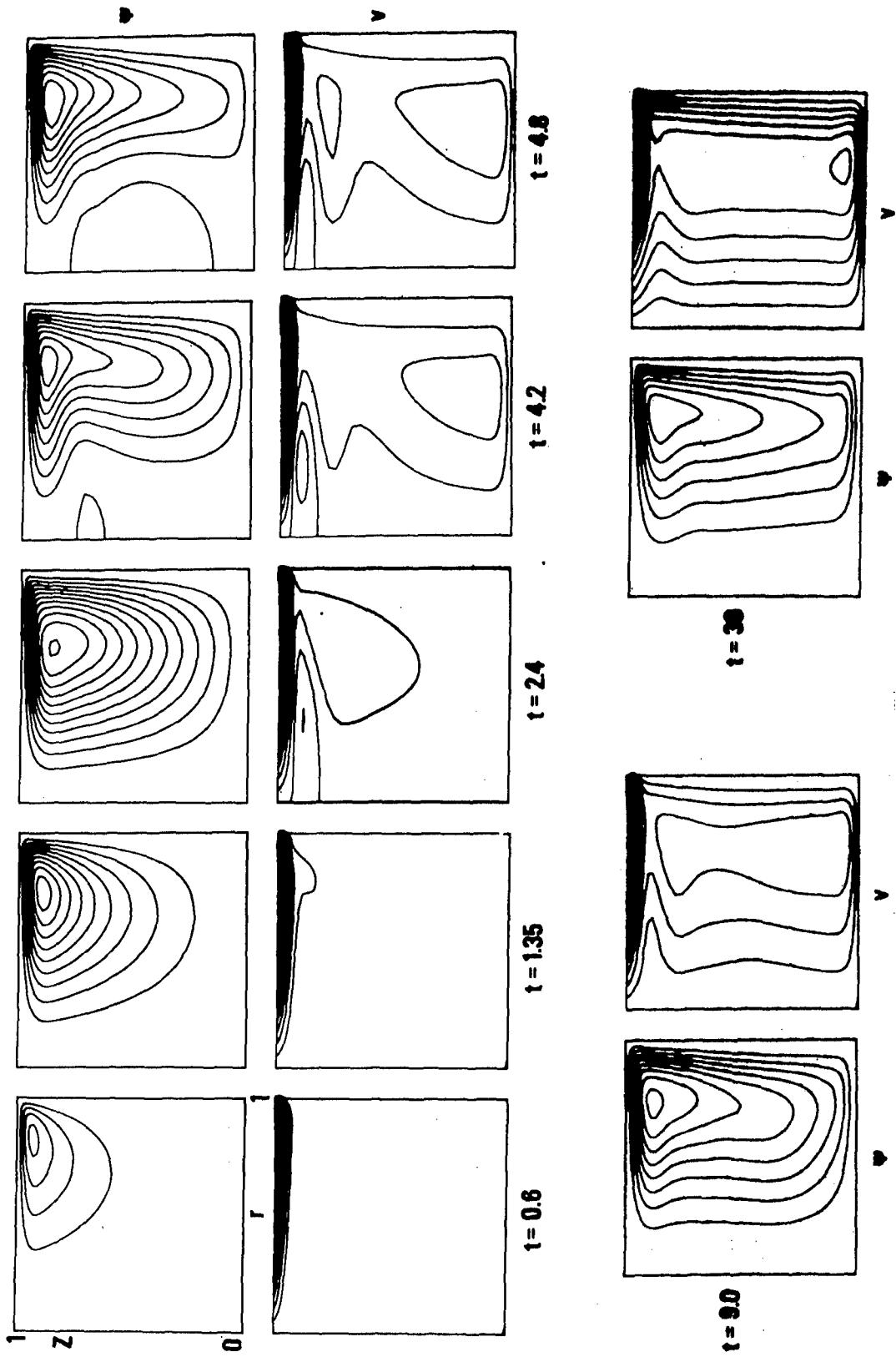


Figure 19 - Lines of Constant ψ and v for $Ro = 1$, $Ek = 0.001$, $\delta = 1$ at Various t . The almost Steady State at $t_{FINAL} = 102$ is Included in Figure 7

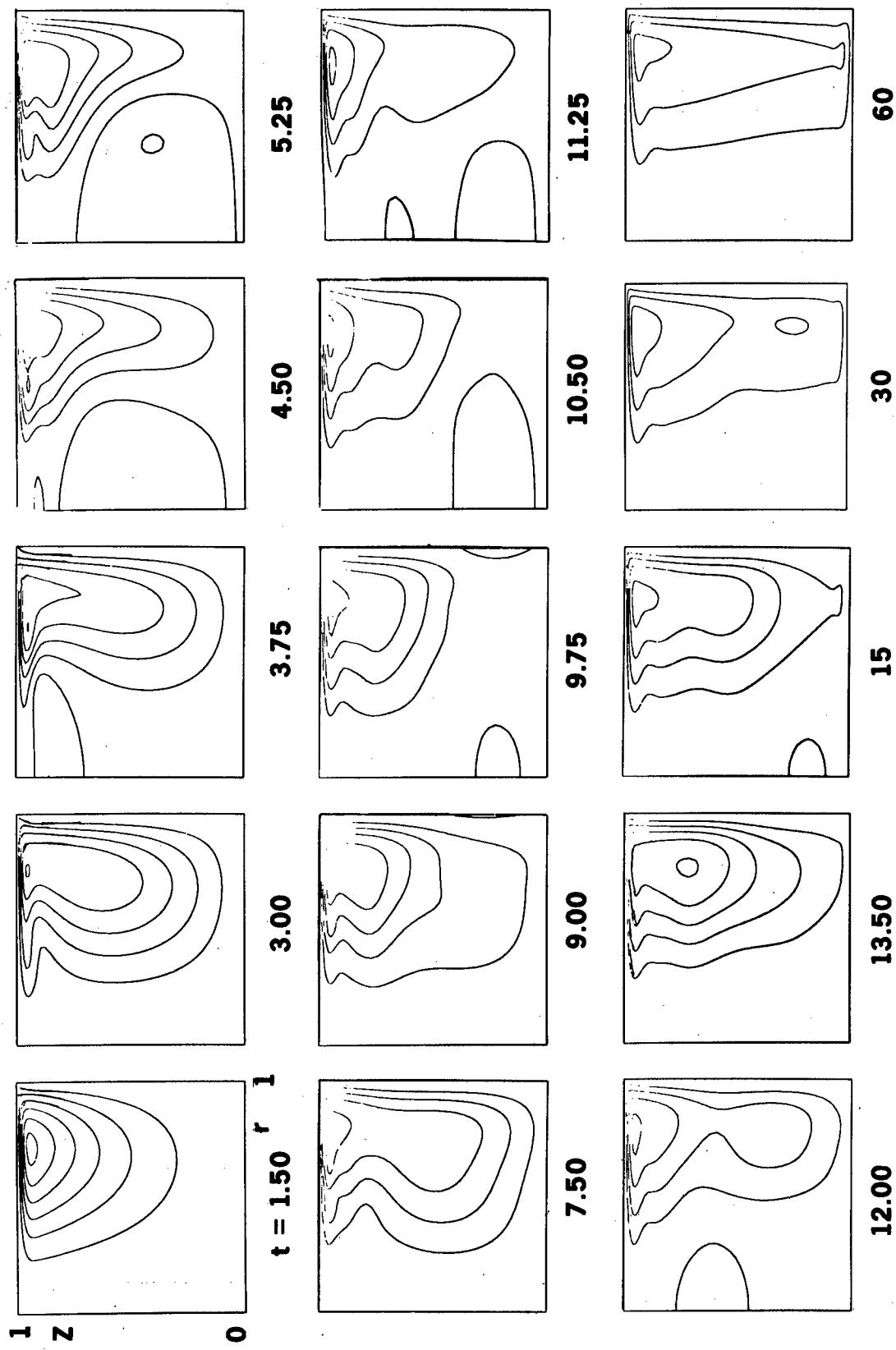


Figure 20 - Lines of Constant ψ for $Ro = 1$, $E_k = 0.0002$ $\delta = 1$ at Various t . The almost Steady State at $t_{FINAL} = 245$ is Included in Figure 8

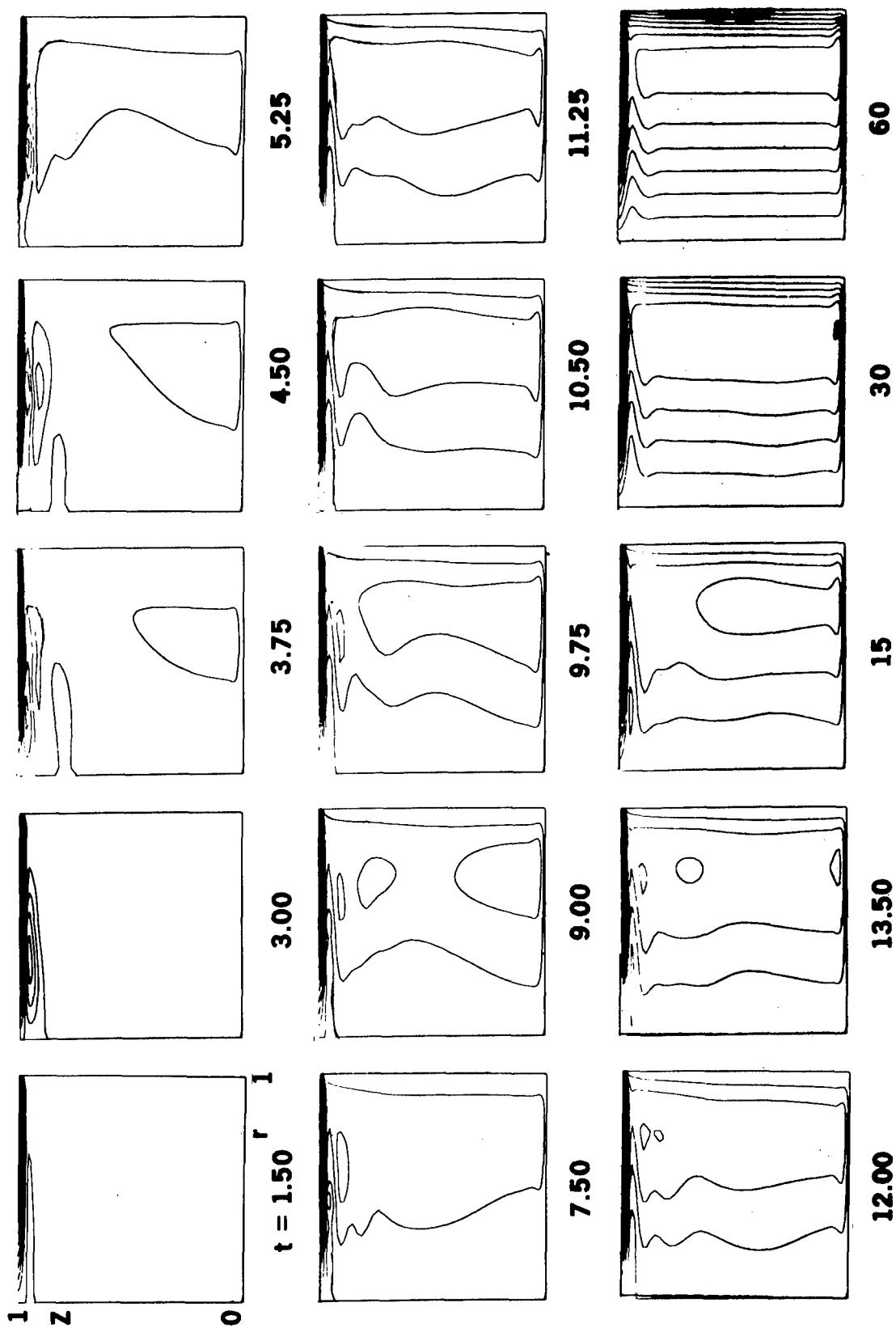


Figure 21 - Lines of Constant v for $Ro = 1$, $Ek = 0.0002$, $\delta = 1$ at Various t . The almost Steady State at $t_{FINAL} = 245$ is Included in Figure 8

Finally, a time sequence of constant ψ , ζ , and v curves is presented for $Ro = 10$, $Ek = 0.01$, $\delta = 1$ in Figure 22. At this high Rossby number the oscillations are shifted toward the sidewall.

5. CONCLUSIONS

Simple geometrical tank configurations with simple initial and boundary conditions for the fluid inside can generate complicated flow circulations. In the transient period inertial waves are visible in the form of oscillating streamlines and cell motions without preferred mode. In the steady-state case monotonic and undulating streamline patterns are distinguished.

The numerical analysis which was developed for nonrotating motion also works well for rotating flows with hyperbolic features. The numerical calculations have been restricted to flows with $Ek \geq 0.0002$ (for $Ro = 1$) since the assumption of axisymmetry does not seem to be justified for smaller Ek .

The linear theory developed in literature on the basis of perturbation methods is restricted for the case $Ro \ll Ek \ll 1$ to $Ek \leq 10^{-3}$ (for $\delta = 1$) and for slow motion according to Reference 3 to $Ek > 0.125$ (for $Ro = 1$, $\delta = 1$).

As $\delta \rightarrow 0$ the computed values approach the similarity solutions for two infinite disks.

The infinitely thin gap between side wall and cover causes a flow singularity which, although local and very weak, results always in an infinite torque. For practical applications a nonvanishing gap width must be considered. The strength of the torque appears to depend crucially on this width (see also Reference 13).

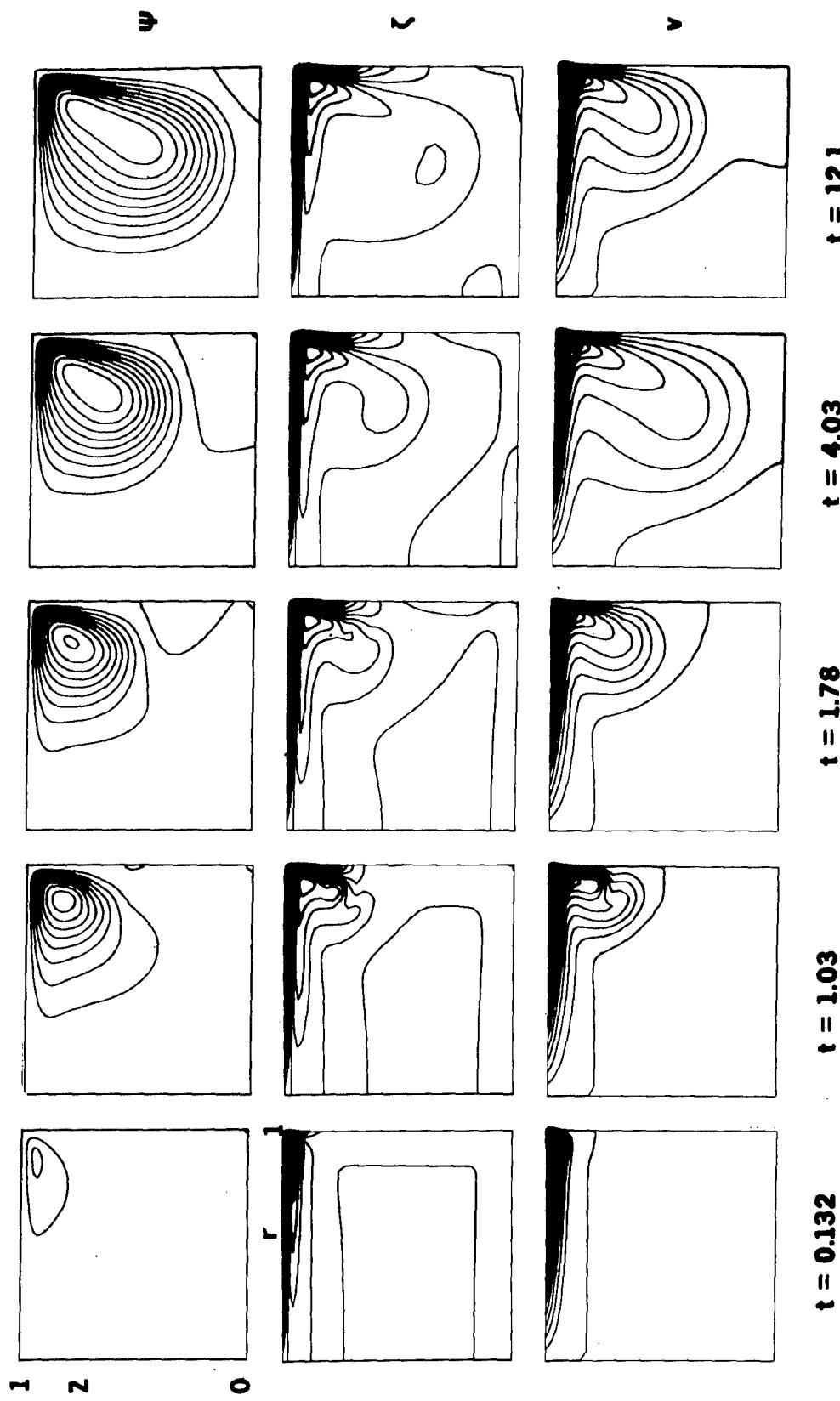


Figure 22 - Lines of Constant Ψ , ζ , and v for $Ro = 10$, $Ek = 0.01$, $\delta = 1$ at Various t . The almost Steady State at $t_{FINAL} = 50$ is Included in Figure 8

ACKNOWLEDGMENT

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13. ABSTRACT A study has been made of axisymmetric incompressible fluid flows in a rotating tank when the angular speed of the cover changes abruptly. From the initial solid-body rotation a meridional and an azimuthal circulation relative to the moving tank develop. This problem is solved numerically by means of a stream function-vorticity formulation for the meridional flow. Local fine grids are used in the Ekman and Stewartson layers. No finite gap between tank and cover is considered. The singular behavior at this point is investigated. The parameters considered are the Rossby number, the Ekman number, and the ratio of height to radius of the tank. Temporal and spatial oscillations of the laminar flow field as well as the occurrence of cell flows are discussed.			

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